Fast Fourier Transform

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Basic FFT

- For a 16-bit FFT, the outputs can be calculated by using the following formula.

\[
y_k = \begin{cases} 
  E_k + e^{-\frac{2\pi i}{N}k} \cdot O_k, & k < N/2 \\
  E_k - \frac{N}{2} - e^{-\frac{2\pi i}{N}(k-N/2)} \cdot O_{k-N/2}, & k \geq N/2
\end{cases}
\]

\[
E_k = \sum_{m=0}^{N-1} x^{2m} e^{-\frac{2\pi i}{N/2}(2m)k} , \quad O_k = \sum_{m=0}^{N-1} x^{2m+1} e^{-\frac{2\pi i}{N/2}(2m)k}
\]
Process Flow

All processes in a stage are carried out concurrently.

Stage 1: 8 Point DFT
- 4 Butterfly Operations

Stage 2: 2 Butterfly Operations

Stage 3: 1 Butterfly Operation
- Butterfly Process
- Butterfly Process

Input: x0, x1, x2, x3, x4, x5, x6, x7
Output: X0, X1, X2, X3, X4, X5, X6, X7

Carried outSequentially
Implementation Details

• **I/O**
  • Input will be sequentially taken from DIP switches.
  • Inputs will be 16 bits: 8 bit real and 8 bit imaginary.
  • Once the processing is done, the output is shown on LED’s.

• **Handling operations**
  • Separately handle real and imaginary parts
  • Create Separate entities for Addition, Subtraction and Multiplication of complex numbers

• **Computing sines and cosines**
  • Lookup Tables will be used to store the values of twiddle factors.
  • Multiples of $e^{-i2\pi n}$ are only stored.
Structural Design

- Each stage will be separated from others by a bank of registers resulting in working of 3 stages.
- All calls to the FFT at each stage can be processed concurrently.
- At each stage $X[k]$ can be divided into even and odd index frequency samples.
- Each stage involves $\frac{N}{2^k}$ such operational butterflies.

\[
A = a + b \\
B = (a - b) \cdot WN
\]
Complex Multiplication Implementation

Real Input

Imaginary Input

Cos + Sin (Twiddle)

Cos - Sin (Twiddle)

Cos (Twiddle)

Imaginary Part

Real Part
Thank You!