Electrical Potential Energy

Nicholas Kortendick
nick@nickkortendick.com

February 8th

Electrical Potential Energy

Electrical Potential Energy is known as voltage. Potential Energy is essentially "stored work". As a mass-filled object falls, gravity is doing work. For electrical potential energy, imagine a setup of two parallel charged plates. For a point particle, \( q \), in a uniform E field, \( U_e = qEd \). This is a force \( \times \) displacement.

Next we will use spherical symmetry. We know gravity’s equation:

\[
U_g = \frac{-GMm}{r}
\]

Thus, \( U = -\int_{\infty}^{r} \frac{-GMm}{r^2} d\vec{r} \). Or more generally:

\[
\Delta U = -\int_{a}^{B} F_E \cdot d\vec{r}
\]

We can use \( U_e = \frac{kQq}{r} \) for our calculations now.

February 11th

Electrical Potential

There’s a slight difference between electrical potential energy and electrical potential. Energy is in terms of work. Electrical potential, also known as voltage, is a scalar. It is the amount of energy that a field provides at a certain space divided by a test charge. The units are therefore (\( \frac{J}{C} \)). Thus the voltage for a point charge is:

\[
Voltage_{point} = \frac{kQq}{r} = \frac{kQ}{r}
\]

Which means:

\[
V = \sum \frac{kQ}{r}
\]

I just learned how to put a hat on my vectors! (\( \vec{yay} \)) Anyways….\( V = -\int \vec{E} \cdot d\vec{r} \) because \( F = -\frac{dv}{dr} \) and \( E = -\frac{dv}{d\vec{r}} \). How can an object be getting kinetic energy when it comes from a place without potential energy? The world may never know..........

Anyways, let’s consider a uniform electrical field:

\[
\Delta U = qEd
\]

\[
\Delta V = \pm Ed
\]
February 13th

Review

Let’s review. The slope between equipotential lines is the electrical field between two lines, thus when electrical fields are closer together, it is more severe. The electrical field lines go down hills. We also have a few equations:

\[ V = \frac{U_q}{Q_0} \]

\[ \Delta V = -\int_a^b \mathbf{E} \cdot d\mathbf{r} \]

\[ \Delta U_e = q \Delta V \]

\[ V = \sum \frac{kq}{r} \]

\[ \Delta V = Ed \]

Integration

If you recall, we used \( E = \int \frac{kdq}{r^2} \) for lines, arcs, hoops, and disks. Today, we will be integrating voltage. Thus, we integrate \( V = \int \frac{kdq}{r} \) for arcs, hoops, and disks. The college board does not require integration of a line.

Arc

Suppose we have an arc of evenly distributed positive charge with radius \( R \). The voltage at the center, by definition, is the amount of energy that a charge is given. Now let’s assume that \( dq \) is a voltage contribution that is \( R \) away. We then need to integrate. Luckily, everything is the same distance away, so it is just a simple sum. The \( V = \frac{ka}{r} \).

Hoop

Now we can integrate a hoop. It is almost as easy to integrate. Say we have the hoop with radius \( R \) and we want to know the voltage at point \( y \) distance away on the principal axis. There is also a total charge \( Q \) on the hoop. If \( dv = \frac{kdq}{r} \). Then,

\[ V = \int \frac{kdq}{\sqrt{R^2 + y^2}} \]

\[ V = \frac{kQ}{\sqrt{R^2 + y^2}} \]

Disk

Finally, let’s examine a disk. A disk will have some charge spread out, and since we are not going to use double integrals, the problem is a little bit tougher. We are using charge density \( \sigma \), radius, \( R \), and upwards distance \( y \). To solve the problem, we will be making a smaller concentric disk with radius \( r \). The electrical field points straight up.

\[ dV = \frac{kdq}{\sqrt{y^2 + r^2}} = \frac{k\sigma2\pi r dr}{\sqrt{y^2 + r^2}} \]

Let \( u = y^2 + r^2 \)
\[ dv = \frac{k \sigma \pi du}{\sqrt{u}} \]
\[ v = k \sigma \pi \int \frac{du}{\sqrt{u}} = 2k \sigma \pi \sqrt{y^2 + r^2} \bigg|_0^R \]

**Sphere**

Imagine a steel bowling ball. The charges will group up evenly on the edge. There is no electrical field on the inside, so there can be no change in voltage inside. There is a plateau on the voltage graph with a downwards line starting at \( R \). Since I’m being extremely lazy right now, it eventually resolves to:

\[ V = \frac{kQ}{R} \]

**January 14th**

**For Future Reference**

Today I’ve decided to start and use smooth draw for some illustrations. Here’s a wonderful picture:

![Squidward](image)

**Anyways....**

Let’s think about two spheres as follows:

![Diagram of two spheres](image)

We would calculate the outside as:

\[ V_a = \frac{2kq}{a} + \frac{1kq}{a} \implies \frac{3kq}{a} \]

We would calculate the middle, between the two spheres, as

\[ V_b = \frac{2kq}{2r} + \frac{kq}{b} \]
We would then calculate the inside as:

\[ V_c = \frac{2kq}{2r} + \frac{kq}{r} \]

To move a 3 coulomb charge from B to A, it requires 6 joules: \((6 - 4) \times 3\). Additionally, the E field is the strongest at A, where it is the steepest.

If \( E = 3x^2 \), \( V = -\int E = -x^3 \).

**January 19th**

**Capacitance**

\[ C = \frac{Q}{V} \]

Capacitance is how much energy something can store per one volt. The units are \( \frac{C}{V} \) or a Farad (F). Because there is a direct relationship between \( Q \) and \( \frac{J}{C} \),

\[ U_c = \frac{1}{2} QV \]

It turns out that a capacitor is purely based on geometric arrangement. The ability for an object to store charge is only geometric based.

**Parallel Plate Capacitor**

If we have two plates, both with area of \( A \) placed distance \( D \) apart, what is the capacitance? Simply, because we can integrate \( V = ED \) to \( V = \frac{Qd}{A\varepsilon_0} \) by using \( \rho \), we get:

\[ C_{||} = \frac{A\varepsilon_0}{d} \]
Spherical Capacitor

Since \( V = -\int_{b}^{a} E \cdot d\vec{r}, \) \( V = -\int_{b}^{a} \frac{kq}{r^2} \cdot dr \), \( V = \frac{-kq}{b} + \frac{kq}{a} = kq\frac{b-a}{ab} \) This resolves to:

\[
C = 4\pi\epsilon_0 \frac{ab}{b-a}
\]
or

\[
C = 4\pi\epsilon R
\]

Cylinder Capacitor

If we consider \( E = \frac{\lambda}{2\pi\epsilon r} \) and substitute in \( Q = \lambda L \) to form \( E = \frac{Q}{2\pi\epsilon_0rL} \), we produce:

\[
C = 2\pi\epsilon_0 \frac{L}{ln\left(\frac{b}{a}\right)}
\]

January 20th

Dielectrics

\[
E_{Effective} = \frac{\rho}{k\epsilon_0}
\]

\[
C = \frac{AK\epsilon_0}{d}
\]

February 25th

Parallel Capacitors

\[
Q_{Total} = Q_1 + Q_2
\]

\[
\Delta V_{Bat} = \Delta V_1 = \Delta V_2
\]

\[
C_{EQ} = \frac{Q_{Total}}{V_{Bat}} = \frac{Q_1 + Q_2}{V_{Bat}} = \frac{Q_1}{\Delta V_1} + \frac{Q_2}{\Delta V_2} \implies C_{Eq} = C_1 + C_2 + ...
\]

Series Capacitors

\[
Q_{Total} = Q_1 = Q_2
\]

\[
V_{Bat} = \Delta V_1 + \Delta V_2
\]

\[
C_{EQ} = \frac{Q_{Total}}{V_{Bat}} = \frac{Q_{Total}}{\Delta V_1 + \Delta V_2} \implies \frac{1}{C_{E}Q} = \frac{\Delta V_1 + \Delta V_2}{Q_{Total}} = \frac{\Delta V_1}{Q_1} + \frac{\Delta V_2}{Q_2} = \frac{1}{C_{E}Q} = \frac{1}{C_1} + \frac{1}{C_2} + ...
\]

Product over Sum! When the capacitance is calculated for a parallel section, the capacitance of each component is just a fraction of the total.
February 26th

Review

The electrical field strength is equal to \( QED \)

February 27th

Negative Charge moves up the voltage hill. Inside of a metal sphere, the voltage is at a plateau. So the Voltage inside of a point is \( \frac{kQ}{R_{\text{Sphere}}} \).