Abstract—A major constraint in deployments of resource-limited networks is the energy consumption related to the battery lifetime of the network nodes. To this end, power efficient digital modulation techniques such as On-Off keying (OOK) are highly attractive. In this paper, we propose a novel complete probabilistic description of the Direct Sequence - Coded Division Multiple Access (DS-CDMA) system with random signatures employing OOK modulation is presented. The system scenario considers simultaneously transmitting nodes in Rayleigh fading conditions. Numerical simulations are provided to support the derived results.

I. INTRODUCTION

Minimizing the energy consumption is a crucial task in the deployment of resource-limited networks. On off keying (OOK) has been proposed as an effective technique for these networks [1]–[3]. In this paper, we assume OOK modulation over direct sequence code division multiple access (DS-CDMA) with randomly selected signatures for each node. This modulation, together with the DS-CDMA, is commonly employed in popular wireless sensor networks (WSNs) standards such as IEEE 802.15.4 [4]. Here, we present a novel complete probabilistic description of the system model.

The probabilistic analysis for the Binary Phase Shift Keying (BPSK) modulation over DS-CDMA with random signatures has been presented in the notable work [5] and more recently in [6], [7], while related results concerning upper and lower bounds in the error probability of the aforementioned system and the bit error rate (BER) in multicarrier systems with mutually independent complementary sets of signatures are presented in [8] and [9], respectively. In many cases, it is of interest to employ algorithms that can compute the optimal detection thresholds for different transmit/receive pairs of nodes [1]. To minimize the computations performed in each node, the derivation of closed-form expressions for the average bit error probability (BEP) is required. The threshold selection problem becomes challenging due to the absence of such expressions. Instead, accurate approximations of acceptable computational complexity for the average BEP may be used to serve this goal. In this paper, we propose such an approximation.

To derive the approximation of the average BEP, assumptions have to be imposed on the nature of fast fading and shadowing impairing the communication links between the desired transmitter and receiver and between the interferers and the desired receiver. Here, we consider only fast fading as in [6], [7] and we examine the Rayleigh case [10], [11].

The rest of the paper is organized as follows: Section II presents the system model occurring by using the OOK modulation over DS-CDMA with random signatures. The BEP is approximated analytically in Section III. Some practical considerations referring to the computation of certain necessary moments are discussed in Section IV. Supporting simulations verifying the validity of our analysis are provided in Section V, whereas Section VI concludes the paper.

II. SYSTEM MODEL

Consider the scenario where \( K \) transmitter-receiver pairs of nodes are communicating. The bits of the \( i \)-th transmitter, \( i = 1, \ldots, K \), are transmitted using an OOK modulation: only bits having value one are transmitted over the wireless channel, while no signal is transmitted when the bit is zero. Assuming DS-CDMA, each bit is transmitted via a spreading sequence. Considering the realistic scenario of imperfect synchronization among distributed nodes, the asynchronous version of DS-CDMA is considered. The same fixed bandwidth \( W \) is allocated to each transmitter-receiver pair. Let \( N = T_i/T_c \) be the processing gain, where \( T_i \) and \( T_c \) are the bit and chip durations, respectively. All the communication links in the network are considered narrowband, therefore the fading is assumed to be flat. These assumptions are representative of the IEEE 802.15.4 standard. We will only consider the case of slow flat fading, therefore all channels are considered quasi-static with respect to the duration of two spreading sequences, i.e., two bit (symbol) durations.

To elaborate more the signal model, we assume that the transmitter of the \( i \)-th link transmits the signal [7]

\[
s_i(t) = \sqrt{2p_i}b_i(t)a_i(t)\cos(\omega_c t + \theta_i),
\]

where \( p_i \) represents the transmitted signal power, \( b_i(t) \) is the data signal (data waveform), \( a_i(t) \) the spreading waveform, \( \omega_c \) the carrier frequency and \( \theta_i \) the phase angle of the carrier for the \( i \)-th transmitter\(^1\). The data signal \( b_i(t) \) is a random process

\(^1\)transmitter of the \( i \)-th link
expressed as

\[ b_i(t) = \sum_{k=-\infty}^{+\infty} b_k^{(i)} d(t - kT_b), \quad (2) \]

where \( d(t) = 1, 0 \leq t < T_b \) and \( d(t) = 0 \) otherwise. Additionally, \( b_k^{(i)} \) denotes the \( i \)-th bit of the \( i \)-th transmitter, taking values in \{0, 1\} with probabilities \( \Pr[b_k^{(i)} = 1] = \pi^{(i)} \). \( \Pr[b_k^{(i)} = 0] = 1 - \pi^{(i)} \) for all \( k \). The spreading signal can be expressed as

\[ a_i(t) = \sum_{k=-\infty}^{+\infty} a_k^{(i)} c(t - kT_c), \quad (3) \]

where \( c(t) \) is the chip waveform time-limited to \([0, T_c]\). Usually, the chip waveform is assumed to be normalized so that \( \int_0^{T_c} c^2(t) dt = T_c \), while the \( k \)-th chip of the \( i \)-th user, \( a_k^{(i)} \), takes on values in \{−1, +1\}. All signatures \( \{a_k^{(i)}\} \) are randomly generated in the sense that every chip polarity is determined by flipping an unbiased coin.

Each signal \( s_i(t) \) is transmitted through a flat fading channel. Let \( h_{ij}(t) \) be the wireless channel coefficient associated to the path from the transmitter of the \( i \)-th link to the receiver of the \( j \)-th link. Then, \( h_{ij}(t) \) can be expressed as [7], [11]

\[ h_{ij}(t) = A_{ij} e^{j\phi_{ij}} \delta(t - \tau_{ij}), \quad (4) \]

where \( \delta(t) \) represents the Dirac delta function. The fading random variables \( A_{ij} \)'s are independent and they follow a Rayleigh distribution. Furthermore, \( A_{ij} \)'s correspond to the envelope of complex Gaussian random processes and \( \{\beta_{ij}\}_{i,j=1}^{K} \) correspond to random phases introduced by the channel, considered to be uniformly distributed in \([0, 2\pi]\) \[11\]. Finally, \( \{\tau_{ij}\}_{i,j=1}^{K} \) correspond to propagation delays through the wireless medium and lack of synchronism amongst the transmitters. They are assumed to be random variables uniformly distributed in \([0, T_b]\).

The received signal at the input of the \( j \)-th correlation (matched filter) receiver is

\[ r_j(t) = \sum_{i=1}^{K} s_i(t) \ast h_{ij}(t) + n_j(t) \]

\[ = \sum_{i=1}^{K} \sqrt{2p_i} A_{ij} b_i(t - \tau_{ij}) a_i(t - \tau_{ij}) \cos(\omega_i t + \phi_{ij}) + n_j(t), \quad (5) \]

where \( \ast \) denotes convolution. Easily, \( \phi_{ij} = \theta_i + \beta_{ij} - \omega_i \tau_{ij} \) and is uniformly distributed in \([0, 2\pi]\). Furthermore, \( n_j(t) \) is assumed to be zero mean, white Gaussian noise with two-sided power spectral density \( N_0/2 \). The random variables, defined above, are generated by different physical sources, thus \( \{A_{ij}\}_{i,j=1}^{K}, \{\phi_{ij}\}_{i,j=1}^{K}, \{b_k^{(i)}\}_{i=1}^{K}, \{\tau_{ij}\}_{i=1}^{K} \) are mutually independent [6], [7].

In the next section, we characterize the bit error probability experienced at a receiver node.

### III. Bit Error Probability

In this section, we first develop a novel characterization of the decision statistic at a receiver node and then we give the expression of the BER. Details follow in the sequel.

#### A. Decision Statistic

Due to the basic assumptions and properties of the spread-spectrum multiple access systems, our setup possesses symmetry with respect to the desired receiver. Therefore, we can focus on the received signal of the \( j \)-th receiver. Additionally, only relative time delays and phase angles are important. Thus, without loss of generality we can set \( \phi_{ij} = \tau_{ij} = 0 \). The parameters \( \{\tau_{ij}\}, \{\phi_{ij}\}, i \neq j \) are now the time delays and phase differences of the \( i \)-th transmitter relative to the \( j \)-th transmitter at the \( j \)-th receiver [5], [8].

The decision statistic at the output of the \( j \)-th coherent correlation receiver for the \( k \)-th transmitted bit of the \( j \)-th transmitter, is

\[ Z_k^{(j)} = D_k^{(j)} + I_k^{(j)} + N_k^{(j)}; \quad (6) \]

where \( D_k^{(j)} \) is the information bearing signal for the \( j \)-th receiver, \( I_k^{(j)} \) is the multiple access interference (MAI) due to the presence of multiple transmitting nodes and \( N_k^{(j)} \) is the AWGN noise having zero mean and variance \( N_0 T_b/4 \). Then

\[ D_k^{(j)} = \sum_{i=1}^{K} \sqrt{\frac{p_i}{2}} A_{ij} T_b b_k^{(i)} \]

and

\[ I_k^{(j)} = \sum_{i=1}^{K} \sum_{i \neq j} \sqrt{\frac{p_i}{2}} A_{ij} \left[ b_k^{(i)} R^{(ij)}(\tau_{ij}) + b_k^{(i)} \tilde{R}^{(ij)}(\tau_{ij}) \right] \cos(\phi_{ij}), \quad (8) \]

where \( R^{(ij)}(\tau_{ij}), \tilde{R}^{(ij)}(\tau_{ij}) \) are continuous-time partial correlation functions given by

\[ R^{(ij)}(\tau_{ij}) = \int_{0}^{\tau_{ij}} a_i(t - \tau_{ij}) a_j(t) dt, \]

\[ \tilde{R}^{(ij)}(\tau_{ij}) = \int_{\tau_{ij}}^{T_b} a_i(t - \tau_{ij}) a_j(t) dt \]

and \( 0 \leq \tau_{ij} < T_b \). We denote \( b_k^{(i-1)} R^{(ij)}(\tau_{ij}) + b_k^{(i)} \tilde{R}^{(ij)}(\tau_{ij}) \) by \( W^{(ij)} \), where we neglect the dependence on the subscript \( k \) due to symmetry of our problem with respect to this subscript. The analysis with respect to \( W^{(ij)} \) in the simpler case of BPSK modulation follows from [5] and [7]. In the more involved case of OOK modulation, it has to be extended. The analysis in [5] relies on conditioning with respect to the signature of the desired transmitter, namely the \( j \)-th signature in our context. With this conditioning the authors are able to make independent the random variables that model the multiple access interference from multiple transmitters. Unfortunately, in the case of the OOK modulation, this conditioning is not enough. We have to further condition on \( b_k^{(i)} \) in order to achieve the desired
independence, while in our analysis we maintain the conditioning on $\gamma_{ij} = [\gamma_{ij}/T_e] \sim \mathcal{U}[0,1,\ldots,N-1]$, where $\mathcal{U}$ denotes the uniform distribution. We use the same notation as in [5] for the last conditioning, namely $\gamma_{ij} = \hat{\gamma}$. In the following analysis, we will not explicitly write this conditioning but its presence will be always considered implicit.

First note that we can easily obtain the corresponding expression to (15) in [5] within our context and without conditioning on $i_k^{(i)}, b_k^{(i)}$:

$$W^{(i)} = P^{(i)}(T_e - S^{(i)}) + Q^{(i)}S^{(i)} + X^{(i)}T_e + Y^{(i)}(T_e - 2S^{(i)}),$$

10

where $S^{(i)} = \tau_{ij} - [\tau_{ij}/T_e]T_e \sim \mathcal{U}(0,T_e)$ by modelling the fractional chip displacement of the $i$th transmitter relative to the $j$th transmitter at the $j$th receiver. To obtain this, we let $\tau(t) = 1,0 \leq t < T_e$. Moreover, $P^{(i)}, Q^{(i)}$ are ternary random variables taking values in $\{0,1,\ldots,1\}$ with probabilities $(1-\pi^{(i)}), \pi^{(i)}/2, \pi^{(i)}/2$, respectively. $X^{(i)}$ is a random variable representing the sum of ternary random variables as $P^{(i)}$ or $Q^{(i)}$ in $A^{(i)}$, where $A^{(i)}$ is the set of indices corresponding to chip boundaries without transitions in the signature waveform of the $j$th transmitter. Similarly, $Y^{(i)}$ is a random variable representing the sum of ternary random variables as $P^{(i)}$ or $Q^{(i)}$ in $B^{(i)}$, where $B^{(i)}$ is the set of indices corresponding to chip boundaries with transitions in the signature waveform of the $j$th transmitter. Clearly, $A^{(i)} \cap B^{(i)} = \emptyset$, $A^{(i)} \cup B^{(i)} = \{0,1,\ldots,N-1\}$, $|A^{(i)}| + |B^{(i)}| = N - 1$, while according to our assumptions both $|A^{(i)}|$ and $|B^{(i)}|$ are uniformly distributed on $\{0,1,\ldots,N-1\}$. Here, $C$ denotes the cardinality of the set $C$, $\cap$ the set intersection, $\cup$ the set union and $\emptyset$ the empty set.

In our context, we can define the corresponding random variables given by (12) in [5], denoted by $Z^{(i)}$ and taking values in $\{0,1,\ldots,1\}$ with probabilities $(1-\pi^{(i)}), \pi^{(i)}/2, \pi^{(i)}/2$, respectively. The random variables $P^{(i)}$, $Q^{(i)}$, $X^{(i)}$, $Y^{(i)}$ can be expressed based on the $Z^{(i)}$'s by the same formulas as in [5]. The $Z^{(i)}$'s are uncorrelated in the case of BPSK modulation, but they are generally correlated if OOK modulation is considered. Specifically, $Z^{(i)}$ for $l \in \{0,1,\ldots,\hat{\gamma} - 1\}$ are independent of any $Z^{(i)}$ for $l \in \{\hat{\gamma},\ldots,N - 1\}$ and correlated with $Z^{(N)}$. The random variables $Z^{(i)}$, $l \in \{0,1,\ldots,\hat{\gamma} - 1\}$ are mutually correlated. The same holds for $Z^{(i)}$, $l \in \{\hat{\gamma},\ldots,N - 1\}$. To understand this point, observe, e.g., that $Pr(Z^{(i)} = 0 | Z^{(i)} = 0) = 1$, while $Pr(Z^{(i)} = 0 | Z^{(i)} = 0) = 0$. Nevertheless, it can be easily seen that conditioning on $i^k - 1, b^k$ the random variables $Z^{(i)}$ become independent.

According to [5] and transferring the notation into our context $P^{(i)} = Z^{(N)} = b^N_{k-1} a^N_{\hat{\gamma} - 1} a^N_{N - 1}$ and $Q^{(i)} = Z^{(N)} = b^N_{k-1} a^N_{\hat{\gamma} - 1} a^N_{N - 1}$, where $+$ signifies a random variable on which we condition. Focusing, e.g., on $P^{(i)}$ we can see that $Pr(P^{(i)} = 0 | b^N_{k-1} = 0, b^N_k = 0, a^N_{N - 1}) = 1$. Additionally, $Pr(P^{(i)} = \pm 1 | b^N_{k-1} = 0$ or $b^N_k = 1, a^N_{N - 1}) = 1/2$ and $Pr(P^{(i)} = \pm 1 | b^N_{k-1} = 0$ or $b^N_k = 0, a^N_{N - 1}) = 0$.

Clearly, $Q^{(i)}$ has the same statistics. Furthermore, we study in the following the statistics of $X^{(i)}$, while $Y^{(i)}$ can be analyzed in a similar fashion. For $X^{(i)}$ we have:

$$X^{(i)} = \sum_{k \in A^{(i)}} Z^{(i)}_{k} = \sum_{k \in A^{(i)}} Z^{(i)}_{\hat{b}^N_{k-1}} + \sum_{k \in A^{(i)}} Z^{(i)}_{b^N_k},$$

11

where $A^{(i)} = A^{(i)} \cap \{0,1,\ldots,\hat{\gamma} - 1\}$ and $A^{(i)} = A^{(i)} \cap \{\hat{\gamma},\ldots,N - 2\}$. Let $\bar{a}^{(i)}$ signify the given $j$th signature sequence. Based on (11), we obtain the probabilistic description of $X^{(i)}$ as follows:

$$Pr\left(X^{(i)} = 0 | b^N_{k-1} = 0, b^N_k = 0, \bar{a}^{(i)}\right) = 1$$

and

$$Pr\left(X^{(i)} \neq 0 | b^N_{k-1} = 0, b^N_k = 0, \bar{a}^{(i)}\right) = 0.$$
the standard Gaussian approximation (SGA) and the improved Gaussian approximation (IGA). It has been observed that the IGA is much more accurate than the SGA, especially for small K. Furthermore, in [13], Holtzman has proposed a much simpler but still accurate BEP approximation based on the Stirling formula. We will pursue an analogous approximation here for the OOK modulation.

We let \( G^{(i)} = \sqrt{p_s/2} A_{ij} \cos(\phi_{ij}) \) and we define the set \( G^{(j)} = \{ G^{(i_1)}, G^{(i_2)}, \ldots, G^{(i-K)} \} \). Similarly, \( S^{(i)} = \{ S^{(i_1)}, S^{(i_2)}, \ldots, S^{(i-j)}, S^{(i+j)}, \ldots, S^{(i-K)} \} \). Therefore, denoting by \( E[.] \) the expectation operator it can easily be checked that \( E[S^{(i)}] = T_s/2 \), \( E[(S^{(i)})^2] = T_s^2/3 \).

Averaging over \( k \) we have \( E[P^{(i)}] = E[Q^{(i)}] = 0 \), \( E[(P^{(i)})^2] = E[(Q^{(i)})^2] = \pi \), \( E[(X^{(i)}) | B^{(i)}] = E[(X^{(i)})] = 0 \) due to the even symmetry of the involved distributions. Furthermore, \( E[(X^{(i)})^2 | B^{(i)}] \) is given by the expression

\[
(1 - \pi^2) \sum_{\lambda \in A^{(i)}_j} \lambda^2 C \left[ \left( \left| A^{(i)}_j \right| + \lambda \right)/2 \right] \nonumber
\]

\[
(1 - \pi^2) \sum_{\lambda \in A^{(i)}_j} \lambda^2 C \left( \left| A^{(i)}_j \right| + \lambda \right)/2 \nonumber
\]

\[
\pi^2 \sum_{\lambda \in A^{(i)}_j} \lambda^2 Q_{X^{(i)}(\lambda)} \nonumber
\]

and \( E[(Y^{(i)})^2 | B^{(i)}] \) is given by the corresponding similar computations. With these results and based on the analysis in [7], [13] for the IGA, we get \( E[T_k^{(i)} + N_k^{(i)}] G^{(i)}(S^{(i)}, B^{(i)}) = 0 \) and

\[
\theta_j = E \left[ \left( T_k^{(i)} + N_k^{(i)} \right)^2 \left| G^{(i)}(S^{(i)}, B^{(i)}) \right. \right] = \nonumber
\]

\[
\sum_{i=1, i \neq j}^K G^{(j)}(S^{(i)})^2 \left[ \pi \| S^{(i)} \|^2 + (T_c - S^{(i)})^2 \right] \nonumber
\]

\[
+ E \left[ (X^{(i)})^2 | B^{(i)} \right] T_s^2 + E \left[ Y^{(i)} | B^{(i)} \right] (T_e - 2S^{(i)})^2 \right] = \nonumber
\]

\[
N_0 T_b/4. \quad \text{Clearly, } G_k^{(j)} \text{ are zero mean Gaussian random variables with variance } E[A_k^2] p_s/4, \text{ and therefore all the moments, hence the probability density functions, of } \theta_j \text{ are known. This is exploited in IGA. Nevertheless, Stirling formula can provide a tight approximation of the probability of error [13].}

The derivation of the BER distinguishes two cases of error: the decision variable \( b \) is decoded as a zero bit, when a one bit was transmitted; or \( \bar{b} \) is decoded as one, when no bit was transmitted. Conditioning on the transmitted bits of the \( j \)th transmitter, its channel amplitude \( A_{j} \) and \( G^{(j)}, S^{(j)}, B^{(j)} \), we denote these probabilities by \( P_{j|\bar{A}_{jj},G^{(j)},S^{(j)},B^{(j)}} \) and \( P_{j|A_{jj},G^{(j)},S^{(j)},B^{(j)}} \), respectively, where

\[
P_{j|\bar{A}_{jj},G^{(j)},S^{(j)},B^{(j)}} = \text{Pr} \left[ Z_k^{(j)} \geq \delta_j | b_k^{(j)} = 0, A_{jj}, G^{(j)}, S^{(j)}, B^{(j)} \right] = Q \left( \frac{\delta_j}{\sqrt{\theta_j}} \right) \nonumber
\]

(13)

\[
P_{j|A_{jj},G^{(j)},S^{(j)},B^{(j)}} = \text{Pr} \left[ Z_k^{(j)} < \delta_j | b_k^{(j)} = 1, A_{jj}, G^{(j)}, S^{(j)}, B^{(j)} \right] = Q \left( \frac{B_k^{(j)} - \delta_j}{\sqrt{\theta_j}} \right) \nonumber
\]

(14)

where \( \delta_j \) is the decision threshold for \( Z_k^{(j)} \), \( Q(x) = 1/\sqrt{2\pi} \int_x^\infty \exp(-t^2/2) \) is the complementary standard Gaussian distribution function. In (14), we have used the even symmetry of the Gaussian bell. By averaging with respect to \( A_{jj}, G^{(j)}, S^{(j)}, B^{(j)} \), we obtain

\[
P_j(\delta_j) = (1 - \pi) E_{A_{jj},G^{(j)},S^{(j)},B^{(j)}} \left[ Q \left( \frac{\delta_j}{\sqrt{\theta_j}} \right) \right] \nonumber
\]

(15)

To obtain a practically handleable expression for the BER, we resort to the Stirling approximation proposed by Holtzman [13], as we mentioned earlier. Let us define the random variables \( \zeta_k(\delta_j) = \delta_j/\sqrt{\theta_j} \), \( \zeta_j(\delta_j) = (B_k^{(j)} - \delta_j)/\sqrt{\theta_j} \).

If \( \mu_{\zeta_k}(\delta_j) \) and \( \sigma_{\zeta_k}(\delta_j) \) be the mean value and standard deviation of \( \zeta_k \) for \( b = 0, 1 \), respectively, with respect to \( A_{jj}, G^{(j)}, S^{(j)}, B^{(j)} \). Then, by [11], [13] we have

\[
E[Q(\zeta_{ij})] \approx f_0(\delta_j) \frac{2}{3} Q \left( \mu_{\zeta_k}(\delta_j) \right) \nonumber
\]

(16)

Eq. (16) can be used to compute an approximate expression of the BER (15):

\[
P_j(\delta_j) \approx (1 - \pi) f_0(\delta_j) + \pi f_1(\delta_j) \nonumber
\]

(17)

In the next section, we present some results helpful in the evaluation of the last BER approximation.

IV. APPLICATION TO RAYLEIGH CHANNELS

According to the above analysis, the receiving nodes must be able to compute the first two moments of \( \zeta_k, b = 0, 1 \), i.e., their mean values and variances with respect to the magnitudes of the channel coefficients. To this end, we propose an approach that will allow the receiving nodes to easily compute the necessary values.

Assuming only multipath fading, we consider Rayleigh distributed channel magnitudes. Then, a quantity that can be easily seen to appear in the moments of \( \zeta_k, b = 0, 1 \) is \( E[A_{jj}] = \sigma \sqrt{\pi/2} \), where \( \sigma \) is the parameter of the Rayleigh distribution. Another quantity of interest is \( E[A_{jj}]^2 \). If \( A_{jj} \) is
Rayleigh distributed, then $A_{ij}^2$ is exponentially distributed and $E[A_{ij}^2] = 2\sigma^2$.

Two other quantities that emerge are $E[1/\sqrt{\theta_j}]$ and $E[1/\theta_j]$. To compute them, numerical integrations and summations have to be performed. Clearly, the integrations can be performed using Riemann sums considering essential support intervals for the involved random variables. Nevertheless, we propose here a way to alleviate the computational burden. The following Lemma will be useful:

**Lemma 1**: Given two random variables $m, n$ assume that $n$ has either no mass in a neighborhood of zero or it has support [$\varepsilon, +\infty$], $\varepsilon > 0$. Furthermore, assume that $E[n] < \infty$ and that $Pr\{n < (2 - \varepsilon)E[n]\} = 1$ for some $\varepsilon > 0$. Then, the mean value of the ratio of $m$ and $n$ can be expressed as follows:

(i) 

$$E \left[ \frac{m}{n} \right] = \frac{E[m]}{E[n]} \left( 1 + \sum_{k=1}^{+\infty} (-1)^k E \left[ \frac{(n - E[n])^k}{(E[n])^k} \right] \right),$$  

(18)

if $m,n$ are independent.

(ii) 

$$E \left[ \frac{m}{n} \right] = \frac{E[m]}{E[n]} \left( 1 + \sum_{k=1}^{+\infty} (-1)^k E \left[ \frac{(n - E[n])^k}{(E[n])^k} \right] \right) + \sum_{k=1}^{+\infty} (-1)^k E \left[ \frac{(n - E[n])^k(m - E(m))}{(E[n])^k E(m)} \right],$$  

(19)

if $m,n$ are not independent.

Based on this Lemma we can approximate $E[1/\sqrt{\theta_j}]$ and $E[1/\theta_j]$ by $1/E[\sqrt{\theta_j}]$ and $1/E[\theta_j]$, respectively. In practice, even if $Pr\{\sqrt{\theta_j} < (2 - \varepsilon)E[\sqrt{\theta_j}]\} < 1$ or $Pr\{\theta_j < (2 - \varepsilon)E[\theta_j]\} < 1$, we can still use the aforementioned approximations provided that they perform well in a target signal to interference plus noise ratio (SINR) regime. Additionally, $1/E[\theta_j]$ can be further approximated by $1/\sqrt{E[\theta_j]}$. These approximations will be meaningful in terms of computational complexity only if $E[\theta_j]$ can be easily computed. The last computation can become easy (in closed form) if we assume independence of $\phi_{ij}$ and $\tau_{ij}$, i.e., independence between $\phi_{ij}$ and $S_{ij}$. Based on the term $\beta_{ij}$ in $\phi_{ij}$ and that $\phi_{ij}$ can be considered mod $2\pi$, such an independence assumption is plausible.

With these approximations, 

$$E_{\text{ij}, S_{ij}}[\theta_j|B_{ij}] = \sum_{i=1, i\neq j}^{K} E \left[ A_{ij}^2 \right] P_{ij} \left\{ \frac{2}{3} \pi^{(i)} T_{c}^2 \right\}$$  

$$+ E \left[ X^{(ij)} \right] T_{c}^2$$  

$$+ E \left[ Y^{(ij)} \right] \left[ T_{b}^2 \right]$$  

$$+ \frac{N_0 T_{b}}{4}$$  

(20)

**V. Simulations**

In this section, numerical examples are provided to validate the derived theoretical results.
Average BER

In this paper, we presented a novel complete probabilistic description of the DS-CDMA system with random signatures employing OOK modulation. Approximations of the BEP useful in practical applications were provided. The system scenario considered simultaneously transmitting nodes in Rayleigh fading conditions. Numerical simulations illustrated our results for the probabilistic OOK DS-CDMA receive signal model.

VI. CONCLUSIONS

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