lecture13: Trees

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Largely based on slides by Cinda Heeren
CS 225 UIUC

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Announcements

- mp3 due tonight (mp4 released tonight)
- mt1 “solution party”
  Wednesday (7/3), 10am
- lab_trees due Thursday night (7/4)
- Reminder: no class 7/4
- Friday (7/5) is the last day to drop
- Review on Friday?
More pointers

In linked lists, each Node had one pointer to another Node. What kind of structures can we make if each Node has more than one pointer?

```
template <class T>
struct TreeNode
{
    T data;
    TreeNode* one;
    TreeNode* two;
    TreeNode* three;
    // etc...
};
```
We’ve created a **tree**. Trees are a collection of **vertices** and **edges**. According to Donald Knuth, they are “…the most important nonlinear structure in computer science.”

Trees are also…

1. Directed
2. Acyclic
3. Connected
Find an edge that is not on the longest path in the tree

Which node would be called the root?

How many parents does each vertex have?

Which vertex has the fewest children?

Which vertex has the most ancestors/descendants?

List all the nodes in C’s subtree

List all the leaves
They have many theoretical elbows.
Abstract Syntax Tree

```
(defun fib (fn (x) (if (< x 2) (if (= x 1) 1 0) (+ (fib (- x 1)) (fib (- x 2))))))
```

Source: http://www.curransoft.com/code/fib_visual_ast_20090414.png
What else?
A binary tree $T$ is either

- empty, $T = {}$
- a root with subtrees, $T = \{ r, T_{\text{left}}, T_{\text{right}} \}$, where $T_{\text{left}}, T_{\text{right}}$ are binary trees

So $T_Z = \{ Z, T_P, T_C \}$ and $T_C = \{ C, T_Q, {} \}$
Height of a tree

The height of a tree is the length of the longest path from the root to a leaf.

- Can you write a recursive definition of $\text{height}(T)$?
- What is the height of the tree to the right?
- What is the height of a tree with only one node?
A full \( n \)-ary tree

A full \( n \)-ary tree is a tree where each node has either 0 or \( n \) children.

- Is the tree to the right a full, binary tree?
- Can you draw one that is?
- What is the height of the smallest full binary tree?
A perfect tree

A perfect tree of height $h$, or $P_h$:

- $P_{-1}$ is an empty tree
- if $h > -1$, then
  
  $P_h = \{ r, T_{left}, T_{right} \}$, where
  $T_{left, right}$ are $P_{h-1}$

Is the tree to the left a perfect tree? What do $P_0$, $P_1$, and $P_2$ look like?

A complete tree has every level full, except possibly the last. Therefore, a perfect tree is a complete tree, but not vice versa.
Quiz!

- What are the minimum number of nodes in a tree of height $h$?
- What are the maximum number of nodes in a tree of height $h$?
- How many nodes are in a perfect tree of height $h$?
- Is every full tree complete?
- Is every complete tree full?
A binary tree implementation

/** @file binary_tree.h */

template <class T>
class BinaryTree
{
    public:
        BinaryTree(); // + big 3
        // ???
    private:
        struct TreeNode
        {
            T data;
            TreeNode* left;       // NULL if no children!
            TreeNode* right;
        };
        TreeNode* root;
};