Abstract – Previous implementations of high frequency oscillators have used either Si-Ge or III-V compound due to the low $f_{\text{max}}$ exhibited in standard CMOS processes. This paper presents and analyzes two commonly used techniques for achieving oscillations above $f_{\text{max}}$. In addition this paper presents a differential push-pull oscillator which is capable of providing a differential output at a frequency above $f_{\text{max}}$.

I. INTRODUCTION

Past circuits implementing sub-millimeter-wave oscillators have used non-standard CMOS based technologies. Previously such frequencies required the use of III-V compounds (such as Ga-As) or a different process such as Si-Ge [1]. At a minimum, these alternate materials require a second process to be included into the design as is the case with Si-Ge. For III-V compounds, the price is also increased in addition to the additional process required. The Tera-Hertz range from 300GHz-3THz has been shown to have uses for radars, sensing applications, imaging, and detection of bio and chemical agents [6]. As such there is a push to get such oscillators working on a standard CMOS process. Several techniques have been used recently in an attempt to get CMOS oscillators working at frequencies beyond $f_{\text{max}}$ of the CMOS process used. The two of these that will be designed and analyzed in this paper are push-pull oscillators [1] [2] [3] [7], and oscillators using a technique called linear-superposition [4]. By showing a CMOS oscillator operating beyond the transistor $f_{\text{max}}$, the possibility of realizing CMOS oscillators in the sub-millimeter-wave range is shown as potentially possible.

II. CROSS-COUPL ED PAIR

The cross-coupled pair is a common topology for implementing differential oscillators. An example of a Cross-Coupled Pair LC oscillator is shown in figure 1. Determining the frequency and start-up requirements for this oscillator are fairly straightforward. $C$ should be broken up into two capacitors both of value 2$C$ going from $V_0+$ and $V_0-$ to ground. As in any LC oscillator, the frequency of oscillation can be shown as:

$$\omega_{\text{osc}} = \frac{1}{\sqrt{L_{\text{eq}}C_{\text{eq}}}} \quad (1)$$

For the circuit shown in figure 1 it will be:

$$\omega_{\text{osc}} = \frac{1}{\sqrt{2LC}} \quad (2)$$

For analyzing the startup requirements it is easy to use Loop-Gain. This circuit is comprised of essentially two common source amplifiers. The loop gain of this circuit then can be shown to be:

$$\text{Loop Gain} = \left(\frac{G_{m}R}{2}\right)^{2} \quad (3)$$

When operating at the frequency of oscillation. In this expression $R$ is the equivalent parallel resistance caused by the inductor and $G_m$ is the transconductance of the transistors. As the inductor $Q$ increases so does the value of $R$ and thus the startup condition for the oscillator becomes easier to meet. This type of oscillator is common among push-pull oscillators due to the ease of making a differential output which is required for push-pull designs. It is this basic circuit that
serves as the fundamental block for both the push-push and the linear-superposition oscillators that are shown in this paper.

III. PUSH-PUSH OSCILLATOR THEORY

Push-Push Oscillators use circuit non-linearities in order to achieve high frequency oscillations. The goal with a push-push design is to add up two opposite waves such that all the odd-order harmonics are cancelled much in the way that a typical differential oscillator would cancel the even order harmonics. The even order harmonics should then be doubled resulting in a fundamental frequency of a push-push oscillator which is twice the value of the original fundamental for the oscillator [1] [2] [3]. This addition first requires the creation of two oscillations of identical frequency and magnitude that vary by 180° in the fundamental. By using a differential oscillator such as the one shown in figure 1, the two out of phase waves are created automatically. What follows is a derivation confirming the theory behind a push-push oscillator.

Taking into account the distortion which is present for any real oscillator, the output voltage of a real oscillator can be represented by f(t).

\[ f(t) = b_0 + \sum_{n=1}^{\infty} b_n \cos(n\omega_0 t) \quad (4) \]

In this equation \( \omega_0 \) represents the fundamental frequency of the oscillator. The sum of two waves that vary by 180° in the fundamental is shown in equation 5.

\[ f_{\text{sum}}(t) = 2b_0 + \sum_{n=1}^{\infty} b_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \cos(n\omega_0 t + \pi) \quad (5) \]

Looking at equation 5, one sees that the 180° phase shift occurs only at the fundamental frequency. All harmonics have a shift that is greater than 180°. This is due to the fact that what really exists is a time shift and thus an identical time shift between the fundamental and the second harmonic would create twice the shift when looking at it in terms of frequency in the second harmonic. This is reflected in equation 5.

Writing out the first four sums from equation 5 results in the following expression:

\[ f_{\text{sum}}(t) = 2b_0 + b_1 [\cos(\omega_0 t) + \cos(\omega_0 t + \pi)] + b_2 [\cos(2\omega_0 t) + \cos(2\omega_0 t + 2\pi)] + b_3 [\cos(3\omega_0 t) + \cos(3\omega_0 t + 3\pi)] + b_4 [\cos(4\omega_0 t) + \cos(4\omega_0 t + 4\pi)] \quad (6) \]

Since \( \cos(x + k\pi) \) is equal to \( -\cos(x) \) for odd values of \( k \) and is equal to \( \cos(x) \) for even powers of \( k \), it is easy to show that equation 6 simplifies to the following expression which contains only even harmonics of the original fundamental.

\[ f_{\text{sum}}(t) = 2b_0 + 2 \sum_{n=1}^{\infty} b_{2n} \cos(2n\omega_0 t) \quad (7) \]

The largest drawback comes from the fact that this method relies entirely on non-linearities in order to generate the harmonics which create the higher frequencies. Typically the harmonics of the fundamental are of a much lower magnitude than the fundamental itself often times yielding results that are much too low to be useful.

IV. LINEAR-SUPERPOSITION THEORY

Linear superposition looks to avoid the major drawback from push-push oscillators in that it looks to create high frequency oscillations that do not rely on the harmonics [4]. This is achieved by the sum of rectified and shifted sinusoidal waves. Figure 2 shows the basic idea of how this would look if four sinusoidal waves were linearly superimposed.

![Figure 2: Linear-superposition [4]](image)

Theoretically if one were able to produce and sum up n equally spaced and rectified sinusoidal waves of equal magnitude they would get an output at a fundamental frequency of \( n\omega_0 \) where \( \omega_0 \) represents the frequency of the
initial waves. To illustrate this, below is the derivation for the case of four evenly spaced waves.

The Fourier series of a perfect rectified sinusoidal wave is given as follows:

\[ f(t) = \frac{A}{\pi} + \frac{A}{2} \sin \omega_0 t - \frac{2A}{\pi} \sum_{n=1}^{\infty} \cos 2n\omega_0 t \frac{4n^2 - 1}{4n^2 - 1} \] (8)

In equation 8, \( A \) represents the magnitude of the sinusoidal waves and \( \omega_0 \) represents the fundamental frequency. If we expand this out to add four different equally spaced waves we get the following expression:

\[
f_{\text{sum}}(t) = \frac{4A}{\pi} \left[ \cos(2\omega_0 t) + \cos(2\omega_0 t + \pi) + \cos(2\omega_0 t + 2\pi) + \cos(2\omega_0 t + 3\pi) \right] \]

\[
= \frac{2A}{\pi} \left[ \cos(4\omega_0 t) + \cos(4\omega_0 t + 2\pi) + \cos(4\omega_0 t + 4\pi) + \cos(4\omega_0 t + 6\pi) \right] \\
- \frac{4A}{\pi} \left[ \cos(6\omega_0 t) + \cos(6\omega_0 t + 3\pi) + \cos(6\omega_0 t + 6\pi) + \cos(6\omega_0 t + 9\pi) \right] \\
+ \frac{8A}{\pi} \left[ \cos(8\omega_0 t) + \cos(8\omega_0 t + 4\pi) + \cos(8\omega_0 t + 8\pi) + \cos(8\omega_0 t + 12\pi) \right] \\
+ \cdots
\]

A keen observer can see that this expression reduces to the following much more manageable equation:

\[
f_{\text{sum}}(t) = \frac{4A}{\pi} - \frac{8A}{\pi} \sum_{n=1}^{\infty} \cos 4n\omega_0 t \frac{16n^2 - 1}{16n^2 - 1} \] (10)

As we can see with four perfectly rectified sinusoidal waveforms added together the result should yield nothing until four times the fundamental frequency. Additionally unlike in push-push techniques, this technique does not rely directly upon circuit non-idealities to create the high frequency wave since harmonics from the oscillator are not required to create the wave. One major drawback however is that the desired signal at 4\( \omega_0 \) is not that much larger than the second harmonic at 8\( \omega_0 \). In fact there is only a 4.2x difference which corresponds to a difference of about 12.5dB. This small amount could potentially cause problems.

V. FUNDAMENTAL-FREQUENCY OSCILLATOR

Since the goal of a push-push oscillator is to obtain oscillations at frequencies beyond \( f_{\text{max}} \) it is important to figure out first what \( f_{\text{max}} \) really is. Figure 3 contains a chart listing \( f_{\text{max}} \) for different technologies. From this chart it looks as if \( f_{\text{max}} \) is around 90GHz for a standard 180nm process.

Figure 3: Fmax across various processes [5].

Figure 4: Fundamental Frequency Oscillator
TABLE-1: Fundamental Oscillator Values

<table>
<thead>
<tr>
<th>Component or Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vdd</td>
<td>1.8 V</td>
</tr>
<tr>
<td>Ibias</td>
<td>2 mA</td>
</tr>
<tr>
<td>L</td>
<td>0.2 nH</td>
</tr>
<tr>
<td>(W/L)$_3$, (W/L)$_4$, (W/L)$_5$, (W/L)$_6$, (W/L)$_7$, (W/L)$_8$, (W/L)$_9$, (W/L)$_10$, (W/L)$_11$</td>
<td>2 µm, 0.18 µm</td>
</tr>
</tbody>
</table>

The schematic shown in figure 4 is almost identical to figure 1 except without the capacitor C. This circuit uses the parasitic capacitance of the transistors to serve as the capacitor in the LC tank. Additionally the inductor L has been created in ASITIC and modelled as the following Pi-Model:

![Figure 5: Inductor modelled in ASITIC](image)

By modelling the inductors this way it takes into account both losses in the inductor as well as a maximum frequency in which the inductor would act inductive. For the 180nm process used in designing this circuit the maximum oscillation frequency that could be obtained was around 60GHz as is shown in figures 6 and 7.

![Figure 6: Transient Analysis of Fundamental Oscillator](image)

VI. QUADRATURE-Oscillator

![Figure 7: Spectrum of Fundamental Oscillator](image)

![Figure 8: Quadrature Wave Oscillator](image)

TABLE-2: Quadrature Wave Oscillator Values

<table>
<thead>
<tr>
<th>Component or Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vdd</td>
<td>1.8 V</td>
</tr>
<tr>
<td>Ibias</td>
<td>2 mA</td>
</tr>
<tr>
<td>L</td>
<td>0.2 nH</td>
</tr>
<tr>
<td>(W/L)$_3$, (W/L)$_4$, (W/L)$_5$, (W/L)$_6$, (W/L)$_7$, (W/L)$_8$, (W/L)$_9$, (W/L)$_10$, (W/L)$_11$</td>
<td>2 µm, 0.18 µm</td>
</tr>
</tbody>
</table>

L uses the inductor modelled in figure 5

The quadruple-push oscillator, differential push-push oscillator, and linear-superposition oscillator all require the creation of four waves that are 90° apart. In order to do this a quadrature oscillator must be designed. Fortunately there is a relatively simple way to create such an oscillator using two cross-coupled pair type oscillators. Using quadrature coupling between the two oscillators using transistors M4, M7, M8 and M11 as shown in figure 8, four equally spaced waveforms can...
be created. Figure 9 shows the output waveforms of the quadrature oscillator and spectrum is shown in figure 10.

\[ f_{\text{sum}}(t) = 2b_0 + \sum_{n=1}^{\infty} b_n \cos(n(\omega_0 t + \frac{\pi}{2})) + \sum_{n=1}^{\infty} b_n \cos(n(\omega_0 t + \frac{3\pi}{2})) \]  

By using a quadrature oscillator, the signals shifted by \( \frac{\pi}{2} \) and \( 3\frac{\pi}{2} \) are already readily available. Below is the first 4 expressions for equation 11.

\[ f_{\text{sum}}(t) = 2b_0 + b_1 \cos(\omega_0 t + \frac{\pi}{2}) + b_1 \cos(\omega_0 t + 3\frac{\pi}{2}) + b_2 \cos(2\omega_0 t \pm \pi) + b_2 \cos(2\omega_0 t \pm 3\pi) + b_3 \cos(3\omega_0 t + \frac{3\pi}{2}) + b_3 \cos(3\omega_0 t + \frac{9\pi}{2}) + b_4 \cos(4\omega_0 t \mp 2\pi) + b_4 \cos(4\omega_0 t + 6\pi) \]  

This expression reduces nicely to:

\[ f_{\text{sum}}(t) = 2b_0 + 2 \sum_{n=1}^{\infty} b_{2n} \cos \left( 2n(\omega_0 t + \frac{\pi}{2}) \right) \]  

Equation 13 is at twice the fundamental frequency (like equation 7) of the original signal however it is shifted 180° degrees when referenced to the fundamental signal. Since we already have a quadrature oscillator it is easy to create a differential push-push oscillator by creating an independent push-push oscillator with each of the two cross-coupled pairs and then taking the output as the difference between the two outputs. Figure 11 shows the schematic for the differential push-push oscillator:

<table>
<thead>
<tr>
<th>Component or Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vdd</td>
<td>1.8 V</td>
</tr>
<tr>
<td>Ibias</td>
<td>2 mA</td>
</tr>
<tr>
<td>L1</td>
<td>0.2 nH</td>
</tr>
<tr>
<td>L2</td>
<td>1 nH</td>
</tr>
<tr>
<td>(W/L)1, (W/L)2, (W/L)3</td>
<td>2 μm</td>
</tr>
<tr>
<td>(W/L)4, (W/L)5, (W/L)6</td>
<td>0.18 μm</td>
</tr>
<tr>
<td>(W/L)7, (W/L)8, (W/L)9</td>
<td>1.5 μm</td>
</tr>
<tr>
<td>(W/L)10, (W/L)11, (W/L)12</td>
<td>0.18 μm</td>
</tr>
</tbody>
</table>

L2 uses the inductor modelled in figure 5

**TABLE 3: Differential Push-Push Oscillator Values**
The output frequency and the spectrum for the differential push-push oscillator are shown in figures 12 and 13. From these one can see that a frequency larger than what was achievable with the fundamental frequency oscillator has been achieved and with a 60dB drop-off between the fundamental and its highest magnitude harmonic.

![Figure 12: Transient waveforms of Differential Push-Push Oscillator](image1)

![Figure 13: Spectrum of Differential Push-Push Oscillator](image2)

This plot shows the transient and spectrum of the differential push-push oscillator.

**VII. QUADRUPLE-PUSH OSCILLATOR**

The quadruple push oscillator shown in figure 14 is a simple extension of the differential push-push oscillator. Remembering back to the theory behind a push-push oscillator explained in section 3, if two signals that are 180° apart are added together, all of their odd harmonics will cancel. Their even harmonics will likewise be amplified by 2. The output of our differential push-push oscillator could thus be used to create a quadruple push oscillator in much the same way that a push-push oscillator can be constructed from a typical differential oscillator. Such an oscillator would have a fundamental frequency that is at four times what the original fundamental frequency was since it was created with an oscillator already running at twice the original fundamental. There is one severe drawback to such an oscillator however. As explained in the theory behind the push-push oscillator is that push-push oscillators tend to have small amplitudes since they are amplifying distortion which is often small to begin with. This effect is only magnified when trying to extract the fourth harmonic. As a result the magnitude of the quadruple-push oscillator is the smallest amongst the oscillators that were constructed. This is apparent in figure 15. For this oscillator transmission lines were also used that were not required for the differential push-push oscillator. By using a quarter wave-length short-circuited transmission sized tuned to the desired frequency, the second harmonic of that frequency can be filtered out. This creates a better output spectrum since the second harmonic is usually the greatest. This was not required in the differential push-push since the second harmonic is automatically cancelled by taking a differential output. Figure 15 shows the output of the quadruple-push oscillator.

**TABLE 4: Quadruple Push Oscillator Values**

<table>
<thead>
<tr>
<th>Component or Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vdd</td>
<td>1.8 V</td>
</tr>
<tr>
<td>Ibias</td>
<td>2 mA</td>
</tr>
<tr>
<td>$L_1$</td>
<td>0.2 nH</td>
</tr>
<tr>
<td>$L_2$</td>
<td>1 nH</td>
</tr>
<tr>
<td>$(W/L)_1, (W/L)_2, (W/L)_3$</td>
<td>2 µm, 0.18 µm</td>
</tr>
<tr>
<td>$(W/L)_4, (W/L)_5, (W/L)_6$</td>
<td>1.5 µm, 0.18 µm</td>
</tr>
</tbody>
</table>

$L_1$ uses the inductor modelled in figure 5
$L_2$ is simulated with a Q of 4

---

![Figure 14: Quadruple Push Oscillator](image3)
The last oscillator that was designed and analyzed was a Linear-Superposition based oscillator. The theory behind how this works is discussed in section 4. As stated in that section, linear super-position requires the addition of rectified waves. In order to do this the output voltages from the quadrature oscillator were first translated into currents. Figure 16 shows the transconductor circuit used. The fundamental oscillator used in this oscillator is exactly the same as the quadrature oscillator shown in figure 8 and throughout this paper. Nodes Ip, Im, Qp, and Qm reference the same nodes as shown in figure 8. The circuit in figure 16 is responsible both for converting the voltages to currents and doing the rectification [4]. Using the quarter-wave length transmission line, the current at the harmonic we desire can only flow through ZL. This creates an output voltage at four times the fundamental frequency as explained in section 4. One big advantage to this method is that each stage in the transconductor sees a current only at the fundamental frequency and the only device which must handle the higher frequency is the load.

Figures 17 and 18 show the output of our oscillator. A couple of our predictions are shown to be valid. First, the magnitude of the linear-superposition based oscillator is much greater than the push-push oscillator. Secondly the distortion is much worse for this oscillator than using push-push techniques. This was again predicted back in section 4.

**X. CONCLUSION**

Of the three high-frequency oscillators presented and analyzed in this paper, all were able to achieve a fundamental frequency of oscillation greater than \( f_{\text{max}} \). Table 6 shows a summary of the results that were achieved.
### TABLE-6 Oscillator Performance Summary

<table>
<thead>
<tr>
<th>Oscillator Topology</th>
<th>Frequency of Oscillation ($f_{osc}$)</th>
<th>Output Magnitude</th>
<th>Magnitude difference between fundamental and third harmonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental Oscillator</td>
<td>60.5GHz</td>
<td>-296.5 dB</td>
<td>38.2 dB</td>
</tr>
<tr>
<td>Differential Push-Push Oscillator</td>
<td>93.98GHz</td>
<td>-17.01 dB</td>
<td>60 dB</td>
</tr>
<tr>
<td>Quadruple Push Oscillator</td>
<td>187.4GHz</td>
<td>-64dB</td>
<td>52.7 dB</td>
</tr>
<tr>
<td>Linear Superposition based Oscillator</td>
<td>188.4GHz</td>
<td>-34.43dB</td>
<td>31.86dB</td>
</tr>
</tbody>
</table>

As expected it is possible to construct a differential push-push oscillator from a quadrature oscillator. Additionally a quadruple-push oscillator capable of achieving a frequency at twice the fundamental is possible in simulation although the uses may be limited due to its small magnitude of oscillation.

As Predicted earlier the Linear Superposition based Oscillator is capable of achieving higher magnitudes than the quadruple-push oscillator. Also predicted however is the fact that the distortion for such an oscillator is much greater than in the push-push based oscillators.

**REFERENCES:**


