This homework is practice only. However, there will be at least one NP-hardness problem on the final exam, so working through this homework is strongly recommended. Students/groups are welcome to submit solutions for feedback (but not credit) in class on May 4, after which we will publish official solutions.

1. Recall that 3SAT asks whether a given boolean formula in conjunctive normal form, with exactly three literals in each clause, is satisfiable. In class we proved that 3SAT is NP-complete, using a reduction from CircuitSAT.

Now consider the related problem 2SAT: Given a boolean formula in conjunctive normal form, with exactly two literals in each clause, is the formula satisfiable? For example, the following boolean formula is a valid input to 2SAT:

\[(x \lor y) \land (y \lor \overline{z}) \land (\overline{x} \lor z) \land (\overline{w} \lor y).\]

Either prove that 2SAT is NP-hard or describe a polynomial-time algorithm to solve it. [Hint: Recall that \((x \lor y) \equiv (\overline{x} \rightarrow y), and build a graph.\]

2. Let \(G = (V,E)\) be a graph. A dominating set in \(G\) is a subset \(S\) of the vertices such that every vertex in \(G\) is either in \(S\) or adjacent to a vertex in \(S\). The DominatingSet problem asks, given a graph \(G\) and an integer \(k\) as input, whether \(G\) contains a dominating set of size \(k\). Either prove that this problem is NP-hard or describe a polynomial-time algorithm to solve it.

A dominating set of size 3 in the Peterson graph.

3. Consider the following solitaire game. The puzzle consists of an \(n \times m\) grid of squares, where each square may be empty, occupied by a red stone, or occupied by a blue stone. The goal of the puzzle is to remove some of the given stones so that the remaining stones satisfy two conditions: (1) every row contains at least one stone, and (2) no column contains stones of both colors. For some initial configurations of stones, reaching this goal is impossible.

Prove that it is NP-hard to determine, given an initial configuration of red and blue stones, whether the puzzle can be solved.