1. Suppose you are given a magic black box that can determine in polynomial time, given an arbitrary graph $G$, the number of vertices in the largest complete subgraph of $G$. Describe and analyze a polynomial-time algorithm that computes, given an arbitrary graph $G$, a complete subgraph of $G$ of maximum size, using this magic black box as a subroutine.

2. **PLANAR CIRCUIT SAT** is a special case of **CIRCUIT SAT** where the input circuit is drawn ‘nicely’ in the plane — no two wires cross, no two gates touch, and each wire touches only the gates it connects. (Not every circuit can be drawn this way!) As in the general **CIRCUIT SAT** problem, we want to determine if there is an input that makes the circuit output **TRUE**?

   **Prove** that **PLANAR CIRCUIT SAT** is NP-complete. *[Hint: XOR]*

3. For each problem below, either describe a polynomial-time algorithm or prove that the problem is NP-complete.

   (a) A **double-Eulerian** circuit in an undirected graph $G$ is a closed walk that traverses every edge in $G$ exactly twice. Given a graph $G$, does $G$ have a **double-Eulerian** circuit?

   (b) A **double-Hamiltonian** circuit in an undirected graph $G$ is a closed walk that visits every vertex in $G$ exactly twice. Given a graph $G$, does $G$ have a **double-Hamiltonian** circuit?