1. Let $G = (V,E)$ be a directed graph with non-negative capacities. Give an efficient algorithm to check whether there is a unique max-flow on $G$?

2. Let $G = (V,E)$ be a graph and $s,t \in V$ be two specific vertices of $G$. We call $(S,T = V\setminus S)$ an $(s,t)$-cut if $s \in S$ and $t \in T$. Moreover, it is a minimum cut if the sum of the capacities of the edges that have one endpoint in $S$ and one endpoint in $T$ equals the maximum $(s,t)$-flow. Show that, both intersection and union of two min-cuts is a min-cut itself.

3. Let $G = (V,E)$ be a graph. For each edge $e$ let $d(e)$ be a demand value attached to it. A flow is feasible if it sends more than $d(e)$ through $e$. Assume you have an oracle that is capable of solving the maximum flow problem. Give efficient algorithms for the following problems that call the oracle at most once.

   (a) Find a feasible flow.

   (b) Find a feasible flow of minimum possible value.