1. For each statement below, check “True” if the statement is always true and “False” otherwise. Each correct answer is worth +1 point; each incorrect answer is worth −½ point; checking “I don’t know” is worth +¼ point; and flipping a coin is (on average) worth +¼ point. You do not need to prove your answer is correct.

Read each statement very carefully. Some of these are deliberately subtle.

(a) If 2 + 2 = 5, then Jeff is not the Queen of England.
(b) For all languages \( L \), the language \( L^* \) is regular.
(c) For all languages \( L \subseteq \Sigma^* \), if \( L \) is can be represented by a regular expression, then \( \Sigma^* \setminus L \) can also be represented by a regular expression.
(d) For all languages \( L_1 \) and \( L_2 \), if \( L_2 \) is regular and \( L_1 \subseteq L_2 \), then \( L_1 \) is regular.
(e) For all languages \( L_1 \) and \( L_2 \), if \( L_2 \) is not regular and \( L_1 \subseteq L_2 \), then \( L_1 \) is not regular.
(f) For all languages \( L \), if \( L \) is not regular, then every fooling set for \( L \) is infinite.
(g) The language \( \{0^m10^n \mid 0 \leq n - m \leq 374 \} \) is regular.
(h) The language \( \{0^m10^n \mid 0 \leq n + m \leq 374 \} \) is regular.
(i) For every language \( L \), if \( L \) is not regular, then the language \( L^R = \{w^R \mid w \in L\} \) is also not regular. (Here \( w^R \) denotes the reversal of string \( w \); for example, \( \text{BACKWARD}^R = \text{DRAWKCAB} \).)
(j) Every context-free language is regular.

2. Let \( L \) be the set of strings in \( \{0, 1\}^* \) in which every run of consecutive \( 0 \)s has odd length and the total number of \( 1 \)s is even.

For example, the string 111000001011000 is in \( L \), because it has eight \( 1 \)s and three runs of consecutive \( 0 \)s, with lengths 5, 1, and 3.

(a) Give a regular expression that represents \( L \).
(b) Construct a DFA that recognizes \( L \).

You do not need to prove that your answers are correct.

3. For each of the following languages over the alphabet \( \{0, 1\} \), either prove that the language is regular or prove that the language is not regular. Exactly one of these two languages is regular.

(a) The set of all strings in which the substrings \( 10 \) and \( 01 \) appear the same number of times.
(b) The set of all strings in which the substrings \( 00 \) and \( 01 \) appear the same number of times.

For example, both of these languages contain the string 1100001101101.
4. Consider the following recursive function:

\[ odds(w) := \begin{cases} 
  w & \text{if } |w| \leq 1 \\
  a \cdot odds(x) & \text{if } w = abx \text{ for some } a, b \in \Sigma \text{ and } x \in \Sigma^* 
\end{cases} \]

Intuitively, \( odds \) removes every other symbol from the input string, starting with the second symbol. For example, \( odds(0101110) = 0010 \).

**Prove** that for any regular language \( L \), the following languages are also regular.

(a) \( \text{ODDS}(L) := \{ odds(w) \mid w \in L \} \).
(b) \( \text{ODDS}^{-1}(L) := \{ w \mid odds(w) \in L \} \).

5. Recall that string concatenation and string reversal are formally defined as follows:

\[
\begin{align*}
  w \cdot y & := \begin{cases} 
    y & \text{if } w = \epsilon \\
    a \cdot (x \cdot y) & \text{if } w = ax \text{ for some } a \in \Sigma \text{ and } x \in \Sigma^* 
  \end{cases} \\
  w^R & := \begin{cases} 
    \epsilon & \text{if } w = \epsilon \\
    x^R \cdot a & \text{if } w = ax \text{ for some } a \in \Sigma \text{ and } x \in \Sigma^* 
  \end{cases}
\end{align*}
\]

**Prove** that \( (w \cdot x)^R = x^R \cdot w^R \), for all strings \( w \) and \( x \). Your proof should be complete, concise, formal, and self-contained. You may assume the following identities, which we proved in class:

- \( w \cdot (x \cdot y) = (w \cdot x) \cdot y \) for all strings \( w, x, \) and \( y \).
- \( |w \cdot x| = |w| + |x| \) for all strings \( w \) and \( x \).