Describe how to simulate an arbitrary Turing machine to make it error-tolerant. Specifically, given an arbitrary Turing machine $M$, describe a new Turing machine $M'$ that accepts and rejects exactly the same strings as $M$, even though an evil pixie named Lenny will move the head of $M'$ to an arbitrary location on the tape some finite number of unknown times during the execution of $M'$.

You do not have to describe $M'$ in complete detail, but do give enough details that a seasoned Turing machine programmer could work out the remaining mechanical details.

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**As stated, this problem has no solution!** If $M$ halts on all inputs after a finite number of steps, then Lenny can make any substring of the input string completely invisible to $M$. For example, if the true input string is *INPUT-STRING*, Lenny can make $M$ believe the input string is actually *IMPING*, by moving the head to the second *I* whenever it tries to move to *R*, and by moving the head to *P* when it tries to move to *U*. Because $M$ halts after a finite number of steps, Lenny only has a finite number of opportunities to move the head.

In fact, with more care, Lenny can make $M$ think the input string is any string that uses only symbols from the actual input string; if the true input string is *INPUT-STRING*, Lenny can make $M$ believe the input string is actually *GRINNING-PUTIN-IS-GRINNING*.)

However, there are several different ways to rescue the problem. For each of the following restrictions on Lenny’s behavior, and for any Turing machine $M$, one can design a Turing machine $M'$ that simulates $M$ despite Lenny’s interference.

- Lenny can move the head only a **bounded** number of times. For example: Lenny can move the head at most 374 times.
- Whenever Lenny moves the head, he changes the state of the machine to a special error state *lenny*.
- Whenever Lenny moves the head, he moves it to the left end of the tape.
- Whenever Lenny moves the head, he moves it to a blank cell to the right of all non-blank cells.
- Whenever Lenny moves the head, he moves it to a cell containing a particular symbol in the input alphabet, say $\theta$.  