1. Recall that $w^R$ denotes the reversal of string $w$; for example, $\text{TURING}^R = \text{GNIRUT}$. Prove that the following language is undecidable.

$$\text{RevAccept} := \{\langle M \rangle \mid M \text{ accepts } \langle M \rangle^R\}$$

2. Let $M$ be a Turing machine, let $w$ be an arbitrary input string, and let $s$ be an integer. We say that $M$ accepts $w$ in space $s$ if, given $w$ as input, $M$ accesses only the first $s$ cells on the tape and eventually accepts.

(a) Prove that the following language is decidable:

$$\{\langle M, w \rangle \mid M \text{ accepts } w \text{ in space } |w|^2\}$$

(b) Prove that the following language is undecidable:

$$\{\langle M \rangle \mid M \text{ accepts at least one string } w \text{ in space } |w|^2\}$$

3. [Extra credit] For each of the following languages, either prove that the language is decidable, or prove that the language is undecidable.

(a) $L_0 = \{\langle M \rangle \mid \text{given any input string, } M \text{ eventually leaves its start state}\}$

(b) $L_1 = \{\langle M \rangle \mid M \text{ decides } L_0\}$

(c) $L_2 = \{\langle M \rangle \mid M \text{ decides } L_1\}$

(d) $L_3 = \{\langle M \rangle \mid M \text{ decides } L_2\}$

(e) $L_4 = \{\langle M \rangle \mid M \text{ decides } L_3\}$
Prove that \( \text{REVACCEPT} := \{ \langle M \rangle \mid M \text{ accepts } \langle M \rangle^R \} \) is undecidable.
(a) Prove that \{ (M, w) \mid M \text{ accepts } w \text{ in space } |w|^2 \} is decidable.

(b) Prove that \{ (M) \mid M \text{ accepts at least one string } w \text{ in space } |w|^2 \} is undecidable:
For each of the following languages, either prove that the language is decidable, or prove that the language is undecidable.

(a) \( L_0 = \{ \langle M \rangle \mid \text{given any input string, } M \text{ eventually leaves its start state} \} \)

(b) \( L_1 = \{ \langle M \rangle \mid M \text{ decides } L_0 \} \)

(c) \( L_2 = \{ \langle M \rangle \mid M \text{ decides } L_1 \} \)

(d) \( L_3 = \{ \langle M \rangle \mid M \text{ decides } L_2 \} \)

(e) \( L_4 = \{ \langle M \rangle \mid M \text{ decides } L_3 \} \)