1. Give regular expressions for each of the following languages over the alphabet \{0, 1\}. You do not need to prove your answers are correct.
   (a) All strings with an odd number of 1s.
   (b) All strings with at most three 0s.
   (c) All strings that do not contain the substring 010.
   (d) All strings in which every occurrence of the substring 00 occurs before every occurrence of the substring 11.

2. Recall that the \textit{reversal} $w^R$ of a string $w$ is defined recursively as follows:
   \[ w^R = \begin{cases} 
   \epsilon & \text{if } w = \epsilon \\
   x \cdot a & \text{if } w = ax \text{ for some symbol } a \text{ and some string } x 
   \end{cases} \]
   The reversal $L^R$ of a language $L$ is defined as the set of reversals of all strings in $L$:
   \[ L^R := \{ w^R \mid w \in L \} \]
   (a) Prove that $(L^*)^R = (L^R)^*$ for every language $L$.
   (b) Prove that the reversal of any regular language is also a regular language. (You may assume part (a) even if you haven’t proved it yet.)
   
   You may assume the following facts without proof:
   \begin{itemize}
   \item $L^* \cdot L^* = L^*$ for every language $L$.
   \item $(w^R)^R = w$ for every string $w$.
   \item $(x \cdot y)^R = y^R \cdot x^R$ for all strings $x$ and $y$.
   \end{itemize}
   \textit{[Hint: Yes, all three proofs use induction, but induction on what? And yes, all three proofs.]}

3. Describe context-free grammars for each of the following languages over the alphabet \{0, 1\}. Explain briefly why your grammars are correct; in particular, describe in English the language generated by each non-terminal in your grammars. (We are not looking for full formal proofs of correctness, but convincing evidence that you understand why your answers are correct.)
   (a) The set of all strings with more than twice as many 0s as 1s.
   (b) The set of all strings that are not palindromes.
   *(c) \textbf{[Extra credit]} The set of all strings that are not equal to $ww$ for any string $w$.
   \textit{[Hint: $a + b = b + a$.]}
Give regular expressions for each of the following languages over the alphabet \{0, 1\}.

(a) All strings with an odd number of 1s.
(b) All strings with at most three 0s.
(c) All strings that do not contain the substring 010.
(d) All strings in which every occurrence of the substring 00 occurs before every occurrence of the substring 11.
(a) Prove that \((L^*)^R = (L^R)^*\) for every language \(L\).

(b) Prove that the reversal of any regular language is also a regular language.
Describe context-free grammars for each of the following languages over the alphabet \{0, 1\}.

(a) The set of all strings with more than twice as many 0s as 1s.
(b) The set of all strings that are not palindromes.
(c) [Extra credit] The set of all strings that are not equal to \(ww\) for any string \(w\).