1. You're organizing the First Annual UIUC Computer Science 72-Hour Dance Exchange, to be held all day Friday, Saturday, and Sunday. Several 30-minute sets of music will be played during the event, and a large number of DJs have applied to perform. You need to hire DJs according to the following constraints.

- Exactly \( k \) sets of music must be played each day, and thus \( 3k \) sets altogether.
- Each set must be played by a single DJ in a consistent music genre (ambient, bubblegum, dubstep, horrorcore, hyphy, trip-hop, Nitzhonot, Kwaito, J-pop, Nashville country, ...).
- Each genre must be played at most once per day.
- Each candidate DJ has given you a list of genres they are willing to play.
- Each DJ can play at most three sets during the entire event.

Suppose there are \( n \) candidate DJs and \( g \) different musical genres available. Describe and analyze an efficient algorithm that either assigns a DJ and a genre to each of the \( 3k \) sets, or correctly reports that no such assignment is possible.

2. Suppose you are given an \( n \times n \) checkerboard with some of the squares deleted. You have a large set of dominos, just the right size to cover two squares of the checkerboard. Describe and analyze an algorithm to determine whether one can tile the board with dominos—each domino must cover exactly two undeleted squares, and each undeleted square must be covered by exactly one domino.

![Checkerboard with dominos](image)

Your input is a two-dimensional array \( \text{Deleted}[1..n, 1..n] \) of bits, where \( \text{Deleted}[i, j] = \text{TRUE} \) if and only if the square in row \( i \) and column \( j \) has been deleted. Your output is a single bit; you do not have to compute the actual placement of dominos. For example, for the board shown above, your algorithm should return \( \text{TRUE} \).

3. Suppose we are given an array \( A[1..m][1..n] \) of non-negative real numbers. We want to round \( A \) to an integer matrix, by replacing each entry \( x \) in \( A \) with either \( \lfloor x \rfloor \) or \( \lceil x \rceil \), without changing the sum of entries in any row or column of \( A \). For example:

\[
\begin{bmatrix}
1.2 & 3.4 & 2.4 \\
3.9 & 4.0 & 2.1 \\
7.9 & 1.6 & 0.5
\end{bmatrix} \quad \rightarrow \quad \begin{bmatrix}
1 & 4 & 2 \\
4 & 4 & 2 \\
8 & 1 & 1
\end{bmatrix}
\]

Describe and analyze an efficient algorithm that either rounds \( A \) in this fashion, or reports correctly that no such rounding is possible.