1. Prove that the following problem is NP-hard.

\textsc{SetCover}: Given a collection of sets \( \{S_1, \ldots, S_m\} \), find the smallest sub-collection of \( S_i \)'s that contains all the elements of \( \bigcup_i S_i \).

2. Given an undirected graph \( G \) and a subset of vertices \( S \), a \textit{Steiner tree} of \( S \) in \( G \) is a subtree of \( G \) that contains every vertex in \( S \). If \( S \) contains every vertex of \( G \), a Steiner tree is just a spanning tree; if \( S \) contains exactly two vertices, any path between them is a Steiner tree.

Given a graph \( G \), a vertex subset \( S \), and an integer \( k \), the \textit{Steiner tree problem} requires us to decide whether there is a Steiner tree of \( S \) in \( G \) with at most \( k \) edges. Prove that the Steiner tree problem is NP-hard. \textit{[Hint: Reduce from VertexCover, or SetCover, or 3Sat.]} 

3. Let \( G \) be a directed graph whose edges are colored red and white. A \textit{Canadian Hamiltonian path} is a Hamiltonian path whose edges are alternately red and white. The \textsc{CanadianHamiltonianPath} problem asks us to find a Canadian Hamiltonian path in a graph \( G \). (Two weeks ago we looked for Hamiltonian paths that cycled through colors on the vertices instead of edges.)

(a) Prove that \textsc{CanadianHamiltonianPath} is NP-Complete.

(b) Reduce \textsc{CanadianHamiltonianPath} to \textsc{HamiltonianPath}.