1. A set of vectors $A$ is said to be linearly independent if no $v \in A$ can be expressed as a linear combination of the vectors in $A - \{v\}$. Given a set of vectors $S$, describe an efficient algorithm for finding a linearly independent subset of $S$ with the maximum possible size. Assume you are given a function that can check if $n$ vectors are linearly independent in $O(n^2)$ time.

2. You live in a country with $n$ different types of coins, with values $1, 2, 2^2, \ldots, 2^{n-1}$. Describe an efficient algorithm for determining how to make change for a given value $W$ using the least possible number of coins.

3. Let $X$ be a set of $n$ intervals on the real line. A proper coloring of $X$ assigns a color to each interval, so that any two overlapping intervals are assigned different colors. Describe an efficient algorithm to compute the minimum number of colors needed to properly color $X$. Assume that your input consists of two array $L[1..n]$ and $R[1..n]$, where $L[i]$ and $R[i]$ are the left and right endpoints of the $i$th interval.