1. An inversion in an array $A[1..n]$ is a pair of indices $(i, j)$ such that $i < j$ and $A[i] > A[j]$. The number of inversions in an $n$-element array is between 0 (if the array is sorted) and $\binom{n}{2}$ (if the array is sorted backward).

Describe and analyze a divide-and-conquer algorithm to count the number of inversions in an $n$-element array in $O(n \log n)$ time. Assume all the elements of the input array are distinct.

2. Suppose you are given two sets of $n$ points, one set $\{p_1, p_2, \ldots, p_n\}$ on the line $y = 0$ and the other set $\{q_1, q_2, \ldots, q_n\}$ on the line $y = 1$. Create a set of $n$ line segments by connect each point $p_i$ to the corresponding point $q_i$. Describe and analyze a divide-and-conquer algorithm to determine how many pairs of these line segments intersect, in $O(n \log n)$ time. [Hint: Use your solution to problem 1.]

Assume a reasonable representation for the input points, and assume the $x$-coordinates of the input points are distinct. For example, for the input shown below, your algorithm should return the number 9.

![Nine intersecting pairs of segments with endpoints on parallel lines.](image)

3. Now suppose you are given two sets $\{p_1, p_2, \ldots, p_n\}$ and $\{q_1, q_2, \ldots, q_n\}$ of $n$ points on the unit circle. Connect each point $p_i$ to the corresponding point $q_i$. Describe and analyze a divide-and-conquer algorithm to determine how many pairs of these line segments intersect in $O(n \log^2 n)$ time. [Hint: Use your solution to problem 2.]

Assume a reasonable representation for the input points, and assume all input points are distinct. For example, for the input shown below, your algorithm should return the number 10.

![Ten intersecting pairs of segments with endpoints on a circle.](image)

4. To think about later: Solve the previous problem in $O(n \log n)$ time.