1. Prove that any non-negative integer can be represented as the sum of distinct powers of 2. (“Write it in binary” is not a proof; it’s just a restatement of what you have to prove.)

2. Prove that any integer can be represented as the sum of distinct powers of $-2$.

3. Write four different proofs that any $n$-node tree has exactly $n - 1$ edges.

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**Take-home points:**

- Induction is recursion. Recursion is induction.
- All induction is strong/structural induction. There is absolutely no point in using a weak induction hypothesis. None. Ever.
- To prove that all snarks are boojums, start with an arbitrary snark and remove some tentacles. Do not start with a smaller snark and try to add tentacles. Snarks don’t like that.
- Every induction proof requires an exhaustive case analysis. Write down the cases. Make sure they’re exhaustive.
- Do the most general cases first. Whatever is left over are the base cases.
- The empty set is the best base case.

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*Khelm is Warsaw. Warsaw is Khelm. Khelm is Warsaw. Zay gezunt!*

*Warsaw is Khelm. Khelm is Warsaw. Warsaw is Khelm. For gezunt!*