1. Suppose you are given an array $A[1..n]$ of positive integers. Describe and analyze an algorithm to find the smallest positive integer that is not an element of $A$ in $O(n)$ time.

2. Suppose you are given an $m \times n$ bitmap, represented by an array $M[1..m, 1..n]$ whose entries are all 0 or 1. A solid block is a subarray of the form $M[i..i', j..j']$ in which every bit is equal to 1. Describe and analyze an efficient algorithm to find a solid block in $M$ with maximum area.
3. Let $T$ be a tree in which each edge $e$ has a weight $w(e)$. A matching $M$ in $T$ is a subset of the edges such that each vertex of $T$ is incident to at most one edge in $M$. The weight of a matching $M$ is the sum of the weights of its edges. Describe and analyze an algorithm to compute a maximum weight matching, given the tree $T$ as input.

4. For any string $x$ and any non-negative integer $k$, let $x^k$ denote the string obtained by concatenating $k$ copies of $x$. For example, STRING$^3$ = STRINGSTRINGSTRING and STRING$^0$ is the empty string.

A repetition of $x$ is a prefix of $x^k$ for some integer $k$. For example, STRINGSTRINGSTRINGST and STR are both repetitions of STRING, as is the empty string.

An interleaving of two strings $x$ and $y$ is any string obtained by shuffling a repetition of $x$ with a repetition of $y$. For example, STRWORINDGSTWORMGRWSTORR is an interleaving of STRING and WORD, as is the empty string.

Describe and analyze an algorithm that accepts three strings $x$, $y$, and $z$ as input, and decides whether $z$ is an interleaving of $x$ and $y$.

5. Every year, as part of its annual meeting, the Antarctican Snail Lovers of Upper Glacierville hold a Round Table Mating Race. Several high-quality breeding snails are placed at the edge of a round table. The snails are numbered in order around the table from 1 to $n$. During the race, each snail wanders around the table, leaving a trail of slime behind it. The snails have been specially trained never to fall off the edge of the table or to cross a slime trail, even their own. If two snails meet, they are declared a breeding pair, removed from the table, and whisked away to a romantic hole in the ground to make little baby snails. Note that some snails may never find a mate, even if the race goes on forever.

For every pair of snails, the Antarctican SLUG race organizers have posted a monetary reward, to be paid to the owners if that pair of snails meets during the Mating Race. Specifically, there is a two-dimensional array $M[1..n, 1..n]$ posted on the wall behind the Round Table, where $M[i, j] = M[j, i]$ is the reward to be paid if snails $i$ and $j$ meet.

Describe and analyze an algorithm to compute the maximum total reward that the organizers could be forced to pay, given the array $M$ as input.