1. An integer program is a linear program with the additional constraint that the variables must take only integer values.

(a) Prove that deciding whether an integer program has a feasible solution is NP-complete.
(b) Prove that finding the optimal solution to an integer program is NP-hard.

[Hint: Almost any NP-hard decision problem can be formulated as an integer program. Pick your favorite.]

2. Describe precisely how to dualize a linear program written in general form:

\[
\begin{align*}
\text{maximize} & \quad \sum_{j=1}^{d} c_j x_j \\
\text{subject to} & \quad \sum_{j=1}^{d} a_{ij} x_j \leq b_i \quad \text{for each } i = 1..p \\
& \quad \sum_{j=1}^{d} a_{ij} x_j = b_i \quad \text{for each } i = p+1..p+q \\
& \quad \sum_{j=1}^{d} a_{ij} x_j \geq b_i \quad \text{for each } i = p+q+1..n
\end{align*}
\]

Keep the number of dual variables as small as possible. The dual of the dual of any linear program should be syntactically identical to the original linear program.

3. Suppose you have a subroutine that can solve linear programs in polynomial time, but only if they are both feasible and bounded. Describe an algorithm that solves arbitrary linear programs in polynomial time, using this subroutine as a black box. Your algorithm should return an optimal solution if one exists; if no optimum exists, your algorithm should report that the input instance is UNBOUNDED or INFEASIBLE, whichever is appropriate. [Hint: Add one constraint to guarantee boundedness; add one variable to guarantee feasibility.]
4. Suppose you are given a set $P$ of $n$ points in some high-dimensional space $\mathbb{R}^d$, each labeled either black or white. A linear classifier is a $d$-dimensional vector $c$ with the following properties:

- If $p$ is a black point, then $p \cdot c > 0$.
- If $p$ is a white point, then $p \cdot c < 0$.

Describe an efficient algorithm to find a linear classifier for the given data points, or correctly report that none exists. [Hint: This is almost trivial, but not quite.]

Lots more linear programming problems can be found at [http://www.ee.ucla.edu/ee236a/homework/problems.pdf](http://www.ee.ucla.edu/ee236a/homework/problems.pdf). Enjoy!