\[ f(u,v,w,x,y) = uv + wxy \]
\[ g(u,v,w,x,y) = uv + wxy' \]

\[ Y = \{ x,y \} \]
\[ f_y^*(u,v,w,x,y) = f(x=0, y=0, v,u,w) = uv + w \]
\[ g_y^*(u,v,w,x,y) = g(u,v,w,x=0, y=0) = uv \]

So residuals are different!

or prove using contradiction

\[ f = X_i f_{x_i} + X_i' f_{x_i} \]
\[ g = X_i g_{x_i} + X_i' g_{x_i} \]

assuming residuals are always equal we get \( f = g \) which is a contradiction.
two variables can be chosen from $m$ in $\binom{m}{2}$ ways.

$m-2$ variables are used to select the residual function.

So the Karnaugh map with $(m-2)$ variables has $2^{m-2}$ squares and each square can be filled with 0, 1, $g(c)$ or $\bar{g}(c)$. So map can be filled in $(4)^{2^{m-2}}$ ways.

$g(c)$ can be formed in $2^4 = 16$ ways and we exclude cases where $g(c) = 0, 1$. So we have $16 - 2 = 14$ cases.

So total decompositions are $\binom{m}{2} \cdot (4)^{2^{m-2}} \cdot 14$.
3. We need to find weights \( \{a_i\} \) and a positive threshold \( T \) such that \( f(x_1, \ldots, x_n) = 1 \) iff
\[
\sum_{i=1}^{n} a_i x_i \geq T
\]
(\( + \) is arithmetic, \( \{x_i\} \) viewed as integers)

<table>
<thead>
<tr>
<th></th>
<th>x_1</th>
<th>x_2</th>
<th>x_3</th>
<th>f</th>
<th>condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>( a_3 \geq T )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( a_2 &lt; T )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>( a_2 + a_3 \geq T )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( a_1 &lt; T )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>( a_1 + a_3 \geq T )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>( a_1 + a_2 \geq T )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>( a_1 + a_2 + a_3 \geq T )</td>
</tr>
</tbody>
</table>

Solution exists, one set might be \( a_3 = 5, a_2 = 3, a_1 = 2, T = 4 \).
4. \[ f(u, v, w, x, y, z) = w'y + u'vw'y' + u'vw'x + uv'wxy \]
\( u : \) no consensus

\( v : \) consensus between \( u'vuv'y' \) and \( u'v'w'x' \)
\( q = u'w'x'y' \), \( D_w = \{w'y'\} \)
\( q \cap D_w = w' \)
\( q \neq q \cap D_w \therefore \) Hazard exists

So input pairs exhibiting this hazard are
(010100, 000100) (010101, 000101)

Product term to add is \( u'w'x'y' \)

\( w : \) consensus between \( w'y \) and \( uv'wxy \)
\( q = uv'x'y \), \( D_w = \{} \)
\( q \cap D_w = uv'x'y \)
\( q \neq q \cap D_w \therefore \) Hazard exists

Input pairs (101111, 100111) (101110, 100110)

Product term to add is \( uv'x'y \)

\( x : \) no consensus
\[ y : \text{consensus between } w'y \text{ and } u'v w' y' \]
\[ q = u'v w' \quad D_y = \{ u'v' w' x' \} \]
\[ q \cap D_y = u' w' \]
\[ q \neq q \cap D_y \implies \text{Hazard exists} \]

Input pairs are: (010001, 010011) (010101, 010111)
(010000, 010010) (010100, 010110)

Product term to add is u'v w'.

\[ z : \text{no consensus} \]