Buy at Bulk Network Design (with Protection)

Chandra Chekuri
Univ. of Illinois, Urbana-Champaign
Optical Network Design

**Goal:** install equipment on network (light up some fibers in dark network) to satisfy (route) traffic

**Objectives:** minimize cost, maximize fault tolerance, ...
Optical Network Design

Details: see tutorial talk by C-Zhang, DIMACS workshop on Next Gen Networks, August 2007
Buy-at-Bulk Network Design

[Salman-Cheriyan-Ravi-Subramanian’97]

Network: graph $G=(V,E)$

Cost functions: for each $e \in E$, $f_e: \mathbb{R}_+ \rightarrow \mathbb{R}_+$

Demand pairs: $s_1t_1, s_2t_2, \ldots, s_ht_h$ (multicommodity)

Demands: $s_it_i$ has a positive demand $d_i$
Buy-at-Bulk Network Design

Feasible solution:
- a multi-commodity flow for the given pairs
- $d_i$ flow from $s_i$ to $t_i$ (can also insist on unsplittable flow along a single path)

Cost of flow: $\sum_e f_e(x_e)$ where $x_e$ is total flow on $e$

Goal: minimize cost of flow
Sink $s$, terminals $t_1, t_2, \ldots, t_h$, demand $d_i$ from $t_i$ to $s$
Economies of scale (sub-additive costs)

\[ f_e(x) + f_e(y) \geq f_e(x+y) \]
Economies of scale (sub-additive costs)

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Economies of scale

- **fixed cost**

- **rent-or-buy**

- **discrete cable capacities**

- **cost-distance (universal)**
Uniform versus Non-uniform

**Uniform:** $f_e = c_e f$ where $c : E \rightarrow \mathcal{R}^+$

(wlog $c_e = 1$ for all $e$, then $f_e = f$)

**Non-uniform:** $f_e$ different for each edge

(can assume wlog is a simple cost-distance function)

Throughout talk graphs are *undirected*
### Approximability

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<tr>
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<th>Single-cable</th>
<th>Uniform</th>
<th>Non-Uniform</th>
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<tbody>
<tr>
<td><strong>Single Source</strong></td>
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<tr>
<td>(hardness)</td>
<td>$O(1)$</td>
<td>$O(1), 20.42$</td>
<td>$O(\log h)$</td>
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<td>$[\text{SCRS' 97}]$</td>
<td>$[\text{GMM'01, GR'10}]$</td>
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<td>$\Omega(1)$</td>
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<td>folklore</td>
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<td><strong>Multicommodity</strong></td>
<td>$O(\log n)$</td>
<td>$O(\log n)$</td>
<td>$O(\log^4 h)^*$</td>
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<td>(hardness)</td>
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<td>$\Omega(\log^{1/4-\varepsilon} n)$</td>
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*$O(\log^3 n)$ for poly-bounded demands $[\text{KN'07}]$
Easy to state open problems

• Close gaps in the table
• Improved bounds for planar graphs or geometric instances?
Three algorithms for Multi-commodity BatB

- Using tree embeddings of graphs for *uniform case*. [Awerbuch-Azar’97]

- Greedy routing with randomization and inflation [Charikar-Karagiazoa’05]

- Junction based approach [C-Hajiaghayi-Kortsarz-Salavatipour’06]
Alg1: Using tree embeddings

Suppose $G$ is a tree $T$

Routing is unique/trivial in $T$

For each $e \in T$, routing induces flow of $x_e$ units

Cost = $\sum_e c_e f(x_e)$

Essentially an optimum solution modulo computing $f$
Alg1: Using tree embeddings

[Bartal’96,’98, FRT’03]

**Theorem:** $O(\log n)$ distortion for embedding a $n$ point finite metric into random dominating tree metrics

[Awerbuch-Azar’97]

**Theorem:** $O(\log n)$ approximation for multicommodity buy-at-bulk with *uniform cost functions*
Open problems for uniform

- Close gap between $O(\log n)$ upper bound and $\Omega(\log^{1/4-\epsilon} n)$ hardness [Andrews'04]
- Obtain an $O(\log h)$ upper bound where $h$ is the number of pairs follows from refinement of tree embeddings due to [Gupta-Viswanath-Ravi’10]
Alg2: Greedy using random permutation

[Charikar-Karagiozova’05] (inspired by [GKRP’03] for rent-or-buy)

Assume $d_i = 1$ for all $i$ // (unit-demand assumption)

Pick a random permutation of demands

// (wlog assume 1,2,...,h is random permutation)

for $i = 1$ to $h$ do

set $d'_i = h/i$ // (pretend demand is larger)

route $d'_i$ for $s_is_i$ greedily along shortest path on current solution

end for
Details

“route $d'_i$ for $s_it_i$ along shortest path on current solution”

$x_j(e)$: flow on $e$ after $j$ demands have been routed

- compute edge costs $c(e) = f_e(x_{i-1}(e)+d'_i) - f_e(x_{i-1}(e))$ // (additional cost of routing $s_it_i$ on $e$)
- compute shortest $s_i-t_i$ path according to $c$
Theorem: Algorithm is $2^{O(\sqrt{\log h \log \log h})}$ approx. for non-uniform cost functions.

Theorem: Algorithm is $O(\log^2 h)$ approx. for non-uniform cost functions in the single-sink case

- Justifies simple greedy algorithm
- Key: randomization and inflation
- Some empirical evidence of goodness
Question/Conjecture: For uniform multi-commodity case, algorithm is $\text{polylog}(h)$ approx.

Question: What is the performance of the algorithm in the non-uniform case? $\text{polylog}(h)$?
Alg3: Junction routing

[HKS’05, CHKS’06]

Junction tree routing:
Alg3: Junction routing

density of junction tree: cost of tree/# of pairs

Algorithm:

While demand pairs left to connect do

• Find a low density junction tree $T$
• Remove pairs connected by $T$
Analysis overview

**OPT**: cost of optimum solution

**Theorem**: In any given instance, there is a junction tree of density $O(\log h) \frac{OPT}{h}$

**Theorem**: There is an $O(\log^2 h)$ approximation for a minimum density junction tree

**Theorem**: Algorithm yields $O(\log^4 h)$ approximation for buy-at-bulk network design
Existence of good junction trees

Three proofs:

1. Sparse covers: $O(\log D) \frac{\text{OPT}}{h}$ where $D = \sum_i d_i$

2. Spanning tree embeddings: $\tilde{O}(\log h) \frac{\text{OPT}}{h}$

3. Probabilistic and recursive partitioning of metric spaces: $O(\log h) \frac{\text{OPT}}{h}$
Min-density junction tree

Similar to single-source? Assume we know junction $r$.

Two issues:
- which pairs to connect?
- how do we ensure that both $s_i$ and $t_i$ are connected to $r$?
Min-density junction tree

[CHKS’06]

**Theorem:** $\alpha$ approximation for single-source via natural LP implies an $O(\alpha \log h)$ approximation for min-density junction tree.

Via [C-Khanna-Naor’01] on single-source LP gap, $O(\log^2 h)$ approximation.

Approach is generic and applies to other problems
Alg3: Open Problems

Close gap for non-uniform: $\Omega(\log^{1/2-\epsilon} n)$ vs $O(\log^4 h)$

- [Kortsarz-Nutov’07] improved to $O(\log^3 n)$ for polynomial demands
- Junction tree analysis is with respect to integral solution. What is the integrality gap of the natural LP?
Buy-at-Bulk with Protection

(1+1)-protection in practical optical networks

For each pair $s_i t_i$ send data simultaneously on node disjoint paths $P_i$ (primary) and $Q_i$ (backup)

Protection against equipment/link failures
Buy-at-Bulk with Protection

More generally:

For each pair \( s_i, t_i \) route on \( k_i \) disjoint paths
(edge or node disjoint depending on applications)

Generalize SNDP (survivable network design problem)
[Antonakopoulos-C-Shepherd-Zhang’07]

2-junction scheme for node-disjoint case:
2-junction-Theorem: $\alpha$-approx for single-source problem via natural LP implies $O(\alpha \log^3 h)$ for multi-commodity problem

- junction density proof (only one of the proofs in three can be generalized with some work)
- single-source problem not easy! $O(1)$ for single-cable via clustering arguments
Buy-at-Bulk with Protection

[C-Korula’08]

Single-sink with vertex-connectivity requirements

- \((\log n)^{O(b)}\) for \(b\) cables for \(k=2\) via clustering args.
- \(2^{O(\sqrt{\log h})}\) for any fixed \(k\) for non-uniform case.

Algorithm is greedy inflation. Is it actually better?

[Gupta-Krishnaswamy-Ravi’10]

- \(O(\log^2 n)\) for \(k=2\) (edge-connectivity, uniform multicommodity)
Open problems

- Approximability of single-sink case for $k=2$. $\alpha$ approx. for single-sink implies $O(\alpha \ polylog(n))$ for multi-comm.

- Single-sink for fixed $k>2$. Best is $2^{O(\sqrt{\log h})}$

- Multi-commodity for fixed $k>2$. 
Conclusion

• Buy-at-bulk network design useful in practice and led to several new theoretical ideas

• Algorithmic ideas:
  • application of Bartal’s tree embedding [AA’97]
  • derandomization and alternative proof of tree embeddings [CCGG’98,CCGGP’98]
  • hierarchical clustering for single-source problems [GMM’00, MMP’00,GMM’01]
  • cost sharing, boosted sampling [GKR’03]
  • junction routing scheme [CHKS’06]

• Hardness of approximation:
  • canonical paths/girth ideas for routing problems [A’04]

• Several open problems
Uniform costs: cable model

In practice costs arise due to discrete capacity cables:

Cables of different type: \((c_1, u_1), (c_2, u_2), \ldots, (c_r, u_r)\)

- \(c_i\): cost of cable of type \(i\)
- \(u_i\): capacity of cable of type \(i\)

\[ u_1 < u_2 < \ldots < u_r \text{ and } c_1/u_1 > c_2/u_2 > \ldots > c_r/u_r \]

Can use multiple copies of each cable type

\[ f(x) = \min \text{ cost set of cables of total capacity at least } x \]