Preface

This volume is dedicated to the papers that were presented at the mini-symposium on “Stabilized and Multiscale Finite Element Methods”, organized at the Fifth World Congress on Computational Mechanics. The conference was held at the Technical University in Vienna, Austria, from July 7 to 12, 2002. The scope of the symposium included all aspects of the stabilized and multiscale finite element methods. The papers presented at the symposium included (i) mathematical theory of the stabilized and multiscale finite element methods, (ii) new stabilized formulations, (iii) application of stabilized methods to multiphysics problems, (iv) implementation issues in stabilized and multiscale methods, and (v) large scale computations with stabilized methods.

The underlying philosophy of the stabilized methods is to strengthen the classical variational formulations so that discrete approximations, which would otherwise be unstable, become stable and convergent. The origins of stabilized methods can be traced back to the early 80’s when Hughes and colleagues realized the issue of lack of stability of the Galerkin method for advection-dominated diffusion problems. In order to correct this deficiency in the standard Galerkin approach they introduced the Streamline-Upwind-Petrov-Galerkin (SUPG) method. Soon thereafter Hughes proposed a generalization of the SUPG method for the Stokes flow problem that opened the door to the mixed-field problems by circumventing the Babuska–Brezzi (BB) inf–sup conditions. The SUPG method turned out to be the forerunner of a new class of stabilization schemes, namely the Galerkin/Least-Squares (GLS) stabilization methods. In the GLS method a least-squares form of the residuals that is based on the corresponding Euler-Lagrange equations is added to the Galerkin finite element formulation. In the context of the advection-dominated diffusion phenomenon it leads to (i) stabilization of the advection operator without upsetting consistency or compromising accuracy, and (ii) circumvention of the BB (inf–sup) condition that restricts the use of many convenient interpolations. During the same era Johnson and coworkers presented the analysis of stabilized methods. A general theory of the stabilized methods was developed and success was achieved on a variety of problems. GLS stabilization was soon followed by the Unusual Stabilized Methods introduced by Franca and coworkers. Concurrently, another class of stabilized methods that is based on the idea of augmenting the Galerkin method with virtual bubble functions was introduced by Brezzi and coworkers. In the mid 90’s Hughes revisited the origins of the stabilization schemes from a variational multiscale viewpoint and presented the Variational Multiscale Method. In this method the different stabilization techniques come together as special cases of the underlying sub-grid scale modeling concept. The key idea in the Hughes’ Variational Multiscale method is to perform a mathematical nesting of the fine scales into the coarse scales, thereby providing a robust framework wherein all the important features of the total solution are consistently represented in the computed solution.

The papers presented in this volume address multiscale issues in computational fluid dynamics, application of the variational multiscale method to unstructured meshes, space–time finite element techniques, time integration schemes, and stabilized methods in solid mechanics. The paper by Gravemier, Wall and...
Ramm presents multilevel finite element approximation to the incompressible Navier–Stokes equations. In their work the mathematical scales at level-two result in a stabilized formulation, while dynamic calculation of sub-grid viscosity involves a third-level of approximation. Koobus and Farhat present an application of the variational multiscale method on unstructured 3-D meshes to the problem of turbulent vortex shedding. They develop an economic projection operator for the fine scales for situations wherein the geometry of the computational domain is complex. Tezduyar and Sathe discuss Enhanced-Discretization Space-Time Technique for the incompressible Navier–Stokes equations together with an advection-diffusion equation that traces a scalar interface function in fluid-structure interaction problems. Their technique addresses the issues that arise when the time step required in the structure sub-domain is smaller than that of the fluid sub-domain. The paper by Codina and Soto discusses important topics related to the finite element solutions of the incompressible Navier–Stokes equations. Coutinho and coworkers present stabilized methods and post-processing techniques for the Darcy flow problem.

There are two papers that present stabilized methods for the advection diffusion equation. Burman and Hansbo’s work is on an edge based stabilization technique for the convection–diffusion–reaction equation. A term penalizing the gradient jumps across element boundaries is introduced, and a convergence proof for the stabilized method is presented. The work by Hauke and Valino extends the use of stochastic fields for the Eulerian representation of a chemical component undergoing diffusion, turbulent convection, and chemical reaction to the joint probability distribution function of several species. The resulting system of nonlinear time-dependent partial differential transport equations is then solved with a stabilized finite element method.

There are two papers that deal with the time integrations schemes. Bochev and coworkers address the question of the behavior of transient stabilized problems for small time steps. They present a detailed mathematical analysis which is accompanied by an interpretation of the different effects. Harari’s paper discusses the deleterious effects that arise because of using very small time steps in implicit time integration schemes. It employs the heat equation and the reaction diffusion equation to illustrate the ideas and discusses the stability issue in the context of the generalized Trapezoidal family of algorithms. The paper then presents a procedure, based on Rothe method, to determine specific values for the potential onset of spatial oscillations. The final paper of this volume is by Bischoff and Bletzinger and presents the Discrete Gap method for construction of Reissner–Mindlin plate models.

The literature on stabilized methods has now become enormous and there is a significant presence of the stabilized methods in the arena of computational fluid dynamics and computational solid mechanics. An offspring of the stabilized methods is the emerging multiscale finite element technology that is creating great excitement in the computational mechanics community. With the burgeoning increase in computational power, the trends in computational mechanics are to provide insights into large scale engineering problems. Looking into the future it is clear that computational mechanics will be required to provide a predictive capability to analyze the behavior of a broad class of multi-physics problems that are increasingly nonlinear and non-smooth in nature. We believe that stabilized and multiscale finite element methods hold the promise to these challenges and will serve as the back bone of the new generation of computational codes.

Finally, we would like to thank all the authors that have contributed to the success of this special issue. Their works present the recent advances in the stabilized and multiscale finite element methods.

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